Homework assignment 6a

1. A discrete random variable x takes the values $\{3, 8, 10\}$ with the respective probabilities 0.2, 0.7, and 0.1. Determine the mean, variance, and standard deviation of x.

   Answer:

   $\mathbb{E}[x] = 7.2$

   $\sigma^2 = 4.76$

   $\sigma = 2.18$

x_i	$P[x_i]$	$x_i P[x_i]$	$(x_i - P[x_i])^2 P[x_i]$
3	0.20	0.60	3.53
8	0.70	5.60	0.45
10	0.10	1.00	0.78
	1.0	7.20	4.76

2. Ed Tompkins, the assistant dean of a business school, has applied for the position of dean of the school of business at a much larger university. The salary at the new university has been advertised as $200,000. He has been told by friends within the administration of the larger university that his chances of getting the position are "about 60%." If Ed stays at his current position, his salary next year will be $120,000. Assuming that his friends have accurately assessed his chances of success, what is Ed's expected salary for next year?

   Answer: Let x be Ed's salary next year. The probability distribution of x is:

   $\mathbb{E}[x] = \$168,000$

x_i	$P[x_i]$	$x_i P[x_i]$
$200,000	0.60	120,000
$120,000	0.40	48,000
	1.0	$168,000

3. A music shop is promoting a sale in which the purchaser of a compact disk can roll a die, then deduct a dollar from the retail price for each dot that shows on the rolled die. It is equally likely that the die will come up any integer from 1 through 6. The owner of the music shop pays \$5.00 for each compact disk, then prices them at \$9.00. During this special promotion, what will be the shop's average profit per compact disk sold? What's the standard deviation of the shop's profit?

   Answer: The profit will be revenue less costs. Revenue depends on the number of dots that show up on the rolled die. In particular, revenue will be 9 minus the number of dots that show up on the rolled die. See table below:

   So, the expected profit per sale is:

   $$\mathbb{E}[x] = \$0.5$$

N. of dots	Revenue	x_i Profit	$P[x_i]$	$x_i P[x_i]$
1	$\$8 = \$9 - \$1$	$\$3 = \$8 - \$5$	0.166667	0.50
2	\$7	\$2	0.166667	0.33
3	\$6	\$1	0.166667	0.17
4	\$5	\$0	0.166667	0.00
5	\$4	- \$1	0.166667	-0.17
6	\$3	-\$2	0.166667	-0.33
			1	\$0.50

4. Suppose you play the following game. A gambler tosses a fair coin repeatedly until it comes up heads [this can go on for ever]. If heads appears on the first roll, she pays you \$2. If heads appears on the second throw, she pays you \$4; if on the third, she pays you \$8; if on the fourth, she pays you \$16; and so on doubling the payoff each time. Let x represent the payoff of this game. What's $\mathbb{E}[x]$? How much would you be welling to pay to play this game?

   Answer: The first thing you need to realize in this problem is that it can go on to infinity. The game *does not* stop at four rounds. The random variable x representing the payoffs of this game takes on the values 0, 2 , 2^2, 2^3, 2^4, 2^5, and so on. The table below shows the probability distribution of x. Notice that $\sum_{i=1}^{n} x_i P[x_i] = 1 + 1 + \cdots + 1$ all the way to infinity. Consequently, the expected value of x is $\mathbb{E}[x] = +\infty$

Number of heads	$x =$ payoff	p[x]	$x_i P[x_i]$
1	2	$\frac{1}{2}$	1
2	2^2	$\frac{1}{2^2}$	1
3	2^3	$\frac{1}{2^3}$	1
4	2^4	$\frac{1}{2^4}$	1
5	2^5	$\frac{1}{2^5}$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$
n	2^n	$\frac{1}{2^n}$	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$