Chapter 7: Continuous Probability Distributions

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Continuous Random Variables

Definition

Continuous Probability Distributions (CPD) describe probabilities associated with continuous random variables (CRVs). Recall that CRVs are able to assume *any* of an *infinite* number of values along an interval.

Example

A chemical compound is randomly selected and let $X =$ the pH value. X can be any value between 0 and 14. Note that X can take *infinitely* many values. So, X is a continuous random variable.

The Probability Distribution of a CRV

Probability distributions of continuous random variables are depicted by smooth curves, where probabilities are expressed as areas under the curves.

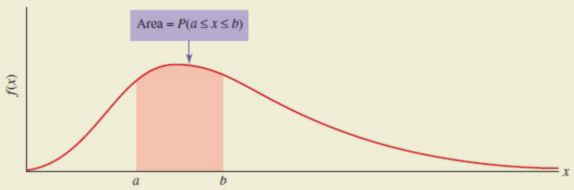
The curve is represented by a function, $f(x)$, referred to as a *probability density function*.

Since the continuous random variable X takes on infinitely many values in any small interval, the probability that X will take on any exact value is regarded as *zero*.

So, when dealing with continuous random variables, we can only speak of the probability that X will be *within* a specified interval of values.

Properties of a Continuous Prob. Dist.

- We plot the range of the CRV on the x axis,
- We plot the probability density function f(x) on the vertical axis.
- The probability that X will take on a value between a and b will be the area under the curve between points a and b.



- The total area under the curve will be equal to 1.

The Continuous Uniform Distribution

Definition

A continuous random variable X is said to have uniform distribution on the interval $[a, b]$ if and only if its probability density function is of the from:

$$f(x) = \begin{cases} \frac{1}{b-a} & \forall x \in [a, b] \\ 0 & \text{Otherwise} \end{cases}$$

The mean and variance of a continuous uniform distribution are:

$$\mathbb{E}[X] = \frac{a+b}{2}$$

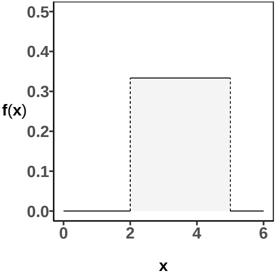
$$\sigma^2 = \frac{(b-a)^2}{12}$$

The Continuous Uniform Dis.: Example

Suppose a bus trip takes between 2 to 5 hours, and that any time within this interval is equally likely to occur. Let X be the time the trip takes.

Since the bus is equally likely to arrive at any moment within the interval $[2, 5]$, the probability density function can be written as:

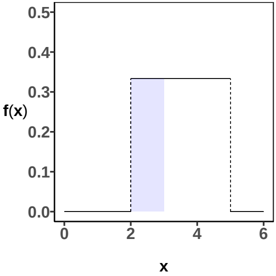
$$f(x) = \begin{cases} \frac{1}{5-2} = \frac{1}{3} & \forall x \in [2, 5] \\ 0 & \text{Otherwise} \end{cases}$$



Notice that the height of the curve is $\frac{1}{3}$, so that the area under the curve equals $\frac{1}{3} \times (5 - 2) = \frac{1}{3} \times 3 = 1$

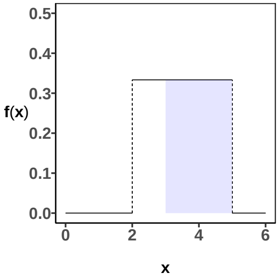
The Continuous Uniform Dist.: Example, contd.

What is $P[2 < X < 3]$?
In this case we are interested in the area under the curve of $f(x)$ that falls within the interval $[2, 3]$.
This is equivalent to the area of the shaded rectangle and it equals $\frac{1}{3} \times 1 = \frac{1}{3}$



The Continuous Uniform Distribution: Example

What is $P[3 < X < 5]$?
In this case we are interested in the area under the curve of $f(x)$ that falls within the interval $[3, 5]$.
This is equivalent to the area of the shaded rectangle and it equals $\frac{1}{3} \times 2 = \frac{2}{3}$



In this case

$$\mathbb{E}[X] = \frac{5+2}{2} = 3.5$$

and

$$\sigma^2 = \frac{(5-2)^2}{12} = \frac{9}{12} = 0.75$$

The Continuous Uniform Dist.: Problem 1

The waiting time at a doctor's office is guaranteed to be less than 20 minutes, and the doctor is equally likely to see patients at any moment during that interval. Let X be the waiting time until a patient is seen by the doctor. Calculate $P[X < 5]$, $P[X > 16]$, $P[10 < X < 18]$, $P[X = 14]$, $P[X > 20]$, and $P[X < 0]$.

Answer:

The Continuous Uniform Dist.: Problem 1

The waiting time at a doctor's office is guaranteed to be less than 20 minutes, and the doctor is equally likely to see patients at any moment during that interval. Let X be the waiting time until a patient is seen by the doctor. Calculate $P[X < 5]$, $P[X > 16]$, $P[10 < X < 18]$, $P[X = 14]$, $P[X > 20]$, and $P[X < 0]$.
Answer: Here $a = 0$, $b = 20$, and:

The Continuous Uniform Dist.: Problem 1

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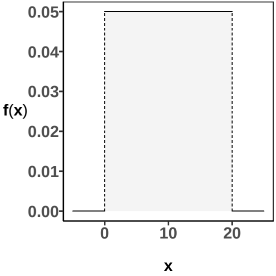
Answer: Here $a = 0$, $b = 20$, and:

$$f(x) = \begin{cases} \frac{1}{20-0} = \frac{1}{20} & \forall x \in [0, 20] \\ 0 & \text{Otherwise} \end{cases}$$



The Continuous Uniform Dist.: Problem 1, Contd.

$P[X < 5] =$

The Continuous Uniform Dist.: Problem 1, Contd.

$P[X < 5] = 5/20$
$P[X > 16] =$

The Continuous Uniform Dist.: Problem 1, Contd.

$P[X < 5] = 5/20$
$P[X > 16] = 4/20$
$P[10 < X < 18] =$

The Continuous Uniform Dist.: Problem 1, Contd.

$P[X < 5] = 5/20$
$P[X > 16] = 4/20$
$P[10 < X < 18] = 8/20$
$P[X = 14] =$

The Continuous Uniform Dist.: Problem 1, Contd.

$P[X < 5] = 5/20$
$P[X > 16] = 4/20$
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$P[X = 14] = 0$
$P[X > 20] =$

The Continuous Uniform Dist.: Problem 1, Contd.

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$P[X < 0] =$

The Continuous Uniform Dist.: Problem 1, Contd.

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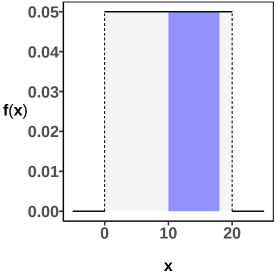
$P[X > 16] = 4/20$
$P[10 < X < 18] = 8/20$
$P[X = 14] = 0$
$P[X > 20] = 0$
$P[X < 0] = 0.$

Figure on the right illustrates the area under the curve for $P[10 < X < 18] = 8/20$



The Continuous Uniform Dist.: Problem 2

The length of smiling time, in seconds, for babies 9 months or younger is a uniformly distributed random variable X that takes on values between 0 and 25 seconds. What is the probability that a randomly chosen baby smiles between 10 and 20 seconds? What is $\mathbb{E}[X]$ and σ_X?

Answer:

The Continuous Uniform Dist.: Problem 2

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Answer: Here $a = 0$, $b = 25$, and:

The Continuous Uniform Dist.: Problem 2

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Answer: Here $a = 0$, $b = 25$, and:

$$f(x) = \begin{cases} \frac{1}{25-0} = \frac{1}{25} & \forall x \in [0, 25] \\ 0 & \text{Otherwise} \end{cases}$$

The Continuous Uniform Dist.: Problem 2

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Answer: Here $a = 0$, $b = 25$, and:

$$f(x) = \begin{cases} \frac{1}{25-0} = \frac{1}{25} & \forall x \in [0, 25] \\ 0 & \text{Otherwise} \end{cases}$$

$P[10 < X < 20] = \frac{10}{25}$

The Continuous Uniform Dist.: Problem 2

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Answer: Here $a = 0$, $b = 25$, and:

$$f(x) = \begin{cases} \frac{1}{25-0} = \frac{1}{25} & \forall x \in [0, 25] \\ 0 & \text{Otherwise} \end{cases}$$

$P[10 < X < 20] = \frac{10}{25}$
$\mathbb{E}[X] = \frac{a+b}{2} = 25/2$

The Continuous Uniform Dist.: Problem 2

The length of smiling time, in seconds, for babies 9 months or younger is a uniformly distributed random variable X that takes on values between 0 and 25 seconds. What is the probability that a randomly chosen baby smiles between 10 and 20 seconds? What is $\mathbb{E}[X]$ and σ_X?

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$$f(x) = \begin{cases} \frac{1}{25-0} = \frac{1}{25} & \forall x \in [0, 25] \\ 0 & \text{Otherwise} \end{cases}$$

$P[10 < X < 20] = \frac{10}{25}$
$\mathbb{E}[X] = \frac{a+b}{2} = 25/2$
$\sigma_X = \frac{(b-a)^2}{12} = 625/12$

The Continuous Uniform Dist.: Problem 2

The length of smiling time, in seconds, for babies 9 months or younger is a uniformly distributed random variable X that takes on values between 0 and 25 seconds. What is the probability that a randomly chosen baby smiles between 10 and 20 seconds? What is $\mathbb{E}[X]$ and σ_X?

Answer: Here $a = 0$, $b = 25$, and:

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$\mathbb{E}[X] = \frac{a+b}{2} = 25/2$
$\sigma_X = \frac{(b-a)^2}{12} = 625/12$

The Normal Distribution

Definition

A continuous random variable X is said to have a ***Normal Distribution*** if and only if its probability density function is of the from:

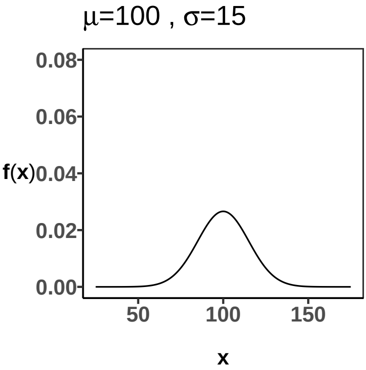
$$f(x) = \frac{1}{\sigma\sqrt{(2\pi)}} e^{-\frac{1}{2}\left[\frac{(X-\mu)}{\sigma}\right]^2}$$

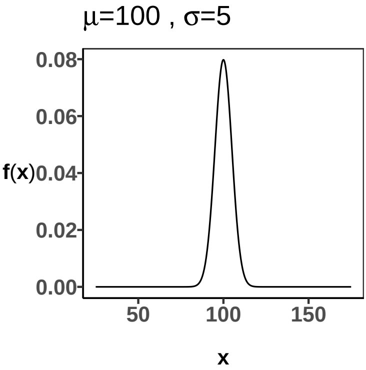
Where $\mathbb{E}[X] = \mu$,
$Var(X) = \sigma^2$,
e is the mathematical constant 2.71828,
and π is the mathematical constant, 3.14159.
We say $X \sim \mathcal{N}(\mu, \sigma)$

For the purposes of this course we do not need to use this expression. It is included here for future reference.

The Shape of the Normal Distribution

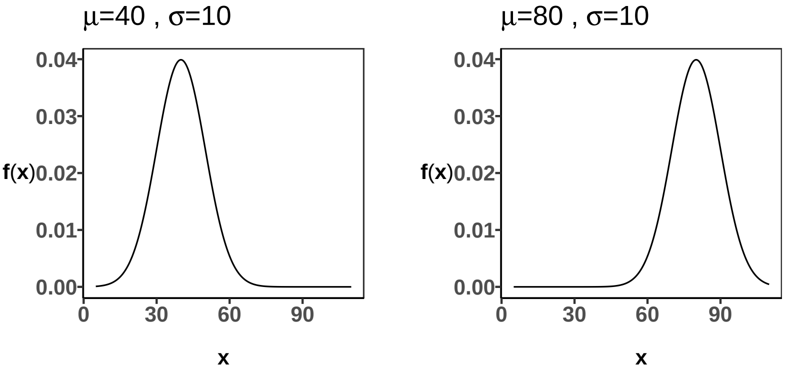
Two distributions with the same mean, but different standard deviations.





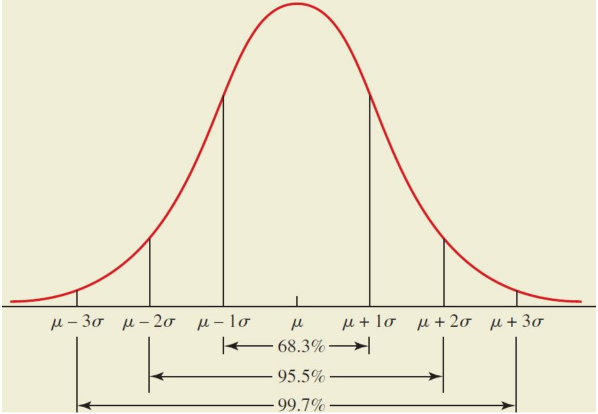
The Shape of the Normal Distribution

Two distributions with the same standard deviation, but different means.



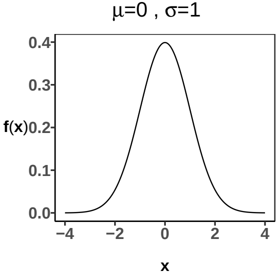
Revisiting the Empirical Rule

Regardless of the shape of the normal distribution, the empirical role will hold.



Introducing the Standard Normal Distribution

- For every pair of μ and σ, the normal distribution $\mathcal{N}(\mu, \sigma)$ has a corresponding shape.
- Recall from chapter 3 that any distribution can be standardized to have a mean of zero and a standard deviation of one.
- When $\mu = 0$ and $\sigma = 1$, we call $\mathcal{N}(0, 1)$ the ***Standard Normal Distribution***.



Standardizing Normal Distributions

Recall from chapter 3 that any variable can be standardized as follows:

$$
\begin{aligned}
z_i = \frac{x_i - \mu}{\sigma} \text{ where } z_i &= \text{standardized value for the } i^{th} \text{ observation} \\
\mu &= \text{the mean} \\
x_i &= \text{the } i^{th} \text{ observation} \\
\sigma &= \text{the standard deviation}
\end{aligned}
$$

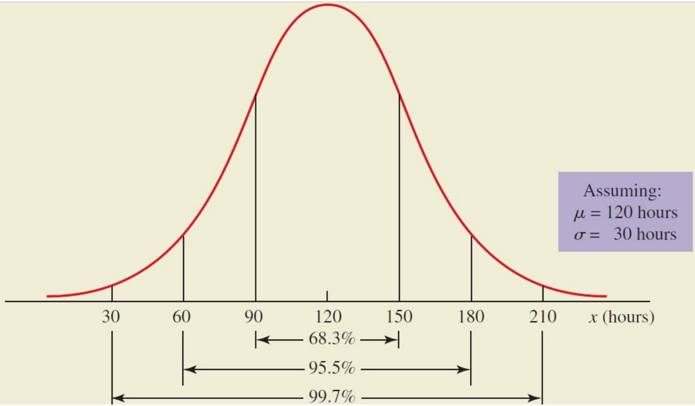
Also, recall that the random variable Z is of mean zero and standard deviation 1.

Any normal distribution can be converted to $\mathcal{N}(0, 1)$.

The Standard Normal Distribution, $\mathcal{N}(0, 1)$, has been ***tabulated***.

Standardizing Normal Dist.: Example 1

According to the General Aviation Manufacturers Association, the annual number of hours flown by a general-aviation aircraft follows $\mathcal{N}(120, 30)$.



Standardizing Normal Dist.: Example 1, Contd.

The annual number of hours flown is a random variable $X \sim \mathcal{N}(120, 30)$. This distribution can be standardized as follows:

- $X = 120 = \mu$ can be standardized to $z = \frac{120-\mu}{\sigma} = \frac{120-120}{30} = 0$
- $X = 30$ can be standardized to

Standardizing Normal Dist.: Example 1, Contd.

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- $X = 30$ can be standardized to $z = \frac{30-120}{30} = -3$
- $X = 60$ can be standardized to

Standardizing Normal Dist.: Example 1, Contd.

The annual number of hours flown is a random variable $X \sim \mathcal{N}(120, 30)$. This distribution can be standardized as follows:

- $X = 120 = \mu$ can be standardized to $z = \frac{120-\mu}{\sigma} = \frac{120-120}{30} = 0$
- $X = 30$ can be standardized to $z = \frac{30-120}{30} = -3$
- $X = 60$ can be standardized to $z = \frac{60-120}{30} = -2$
- $X = 90$ can be standardized to

Standardizing Normal Dist.: Example 1, Contd.

The annual number of hours flown is a random variable $X \sim \mathcal{N}(120, 30)$. This distribution can be standardized as follows:

- $X = 120 = \mu$ can be standardized to $z = \frac{120-\mu}{\sigma} = \frac{120-120}{30} = 0$
- $X = 30$ can be standardized to $z = \frac{30-120}{30} = -3$
- $X = 60$ can be standardized to $z = \frac{60-120}{30} = -2$
- $X = 90$ can be standardized to $z = \frac{90-120}{30} = -1$
- $X = 150$ can be standardized to

Standardizing Normal Dist.: Example 1, Contd.

The annual number of hours flown is a random variable $X \sim \mathcal{N}(120, 30)$. This distribution can be standardized as follows:

- $X = 120 = \mu$ can be standardized to $z = \frac{120-\mu}{\sigma} = \frac{120-120}{30} = 0$
- $X = 30$ can be standardized to $z = \frac{30-120}{30} = -3$
- $X = 60$ can be standardized to $z = \frac{60-120}{30} = -2$
- $X = 90$ can be standardized to $z = \frac{90-120}{30} = -1$
- $X = 150$ can be standardized to $z = \frac{150-120}{30} = 1$
- $X = 180$ can be standardized to

Standardizing Normal Dist.: Example 1, Contd.

The annual number of hours flown is a random variable $X \sim \mathcal{N}(120, 30)$. This distribution can be standardized as follows:

- $X = 120 = \mu$ can be standardized to $z = \frac{120-\mu}{\sigma} = \frac{120-120}{30} = 0$
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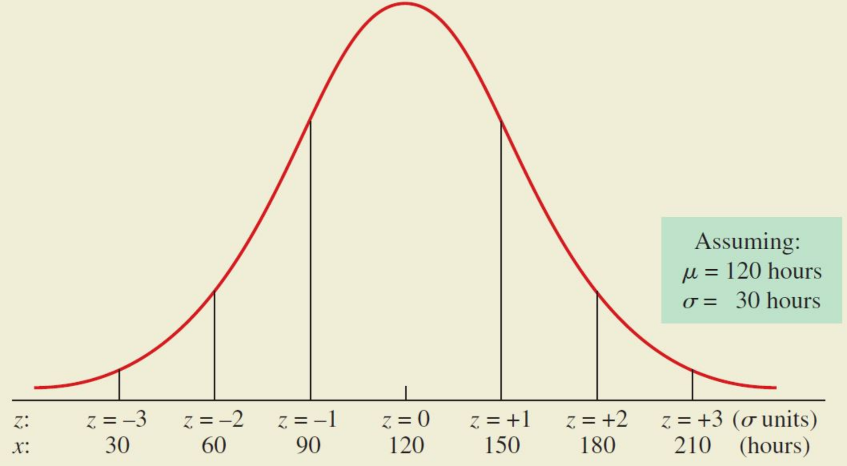
- $X = 90$ can be standardized to $z = \frac{90-120}{30} = -1$
- $X = 150$ can be standardized to $z = \frac{150-120}{30} = 1$
- $X = 180$ can be standardized to $z = \frac{180-120}{30} = 2$
- $X = 210$ can be standardized to

Standardizing Normal Dist.: Example 1, Contd.

The annual number of hours flown is a random variable $X \sim \mathcal{N}(120, 30)$. This distribution can be standardized as follows:

- $X = 120 = \mu$ can be standardized to $z = \frac{120-\mu}{\sigma} = \frac{120-120}{30} = 0$
- $X = 30$ can be standardized to $z = \frac{30-120}{30} = -3$
- $X = 60$ can be standardized to $z = \frac{60-120}{30} = -2$
- $X = 90$ can be standardized to $z = \frac{90-120}{30} = -1$
- $X = 150$ can be standardized to $z = \frac{150-120}{30} = 1$
- $X = 180$ can be standardized to $z = \frac{180-120}{30} = 2$
- $X = 210$ can be standardized to $z = \frac{210-120}{30} = 3$

The new distribution will have the same shape as the old one, but it will be re-scaled such that $\mu = 0$ and $\sigma = 1$



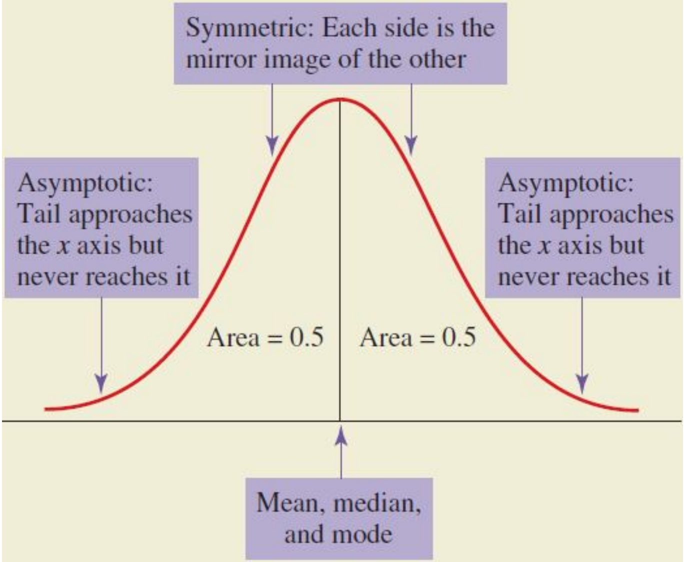
Calculating Probabilities from a Normal Dist.

- To calculate the probabilities, we need to know the area under the normal distribution curve.
- For every pair μ and σ, there is a different normal distribution curve.
- Thus, we would need an infinite number of statistical tables, if we wished to determine the areas corresponding to possible intervals within all of them.
- To solve this problem, we always standardize normal distributions. This allows us to use one table, that of the Standard Normal Distribution $\mathcal{N}(0, 1)$.

Calculating Probabilities from a Normal Dist.: Summary

1. $P[X < x]$ is the area under the curve to the left of x.
2. To calculate $P[X < x]$, start by standardizing X and x.
3. After standardizing X and x, use the standard normal distribution table (Also known as the z table) to find the corresponding probability.

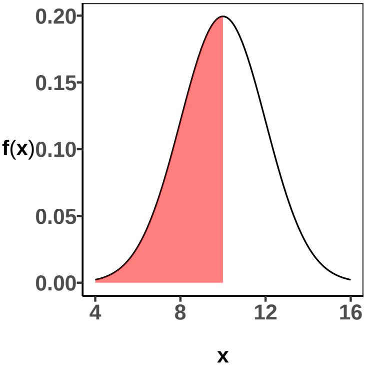
Symmetry of the Normal Distribution



Calculating Normal Dist. Probab.: Example 1

Since the normal distribution is a continuous probability distribution, we calculate the probability that X is less than some value x, $P[X < x]$, by calculating the area under the curve to the left of x.

Suppose $X \sim \mathcal{N}(10, 2)$, what is $P[X < 10]$?



By symmetry, we know that half of the area is to the left of the mean, 10.

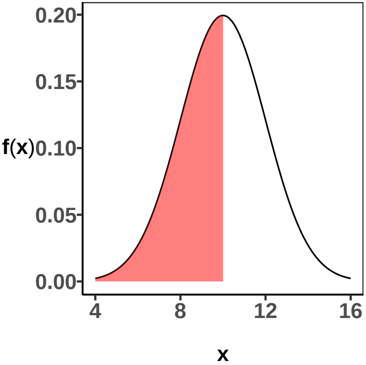
Also, we know that the total area under the curve is 1.

So, $P[X < 10] = 0.5$

Calculating Normal Dist. Probab.: Example 1, Contd.

Alternatively:
Given that $X \sim \mathcal{N}(10, 2)$,
$P[X < 10]$ can be calculated as:

$$P[X < 10] = P[\frac{X-\mu}{\sigma} < \frac{10-\mu}{\sigma}]$$
$$= P[Z < \frac{10-10}{2}]$$
$$= P[Z < 0]$$



Now, look up the Z Table and verify that $[Z < 0] = 0.5$

Introducing the Z table

Table 1: Cumulative standard normal probabilities: $P[Z \leq z]$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133

Calculating Normal Dist. Probab.: Example 2

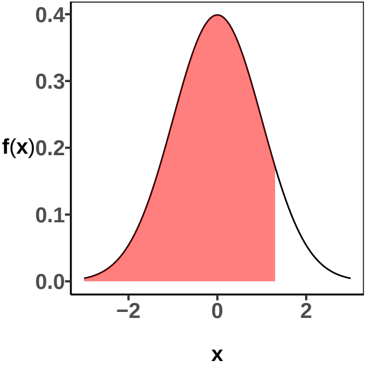
Suppose $X \sim \mathcal{N}(0, 1)$, what is $P[X < 1.3]$?

How can we calculate the area to the left of $X = 1.3$?

Notice that this is already a standard normal distribution.

We can use calculus to evaluate the area under the curve of

$$f(x) = \frac{1}{1\sqrt{(2\pi)}} e^{-\frac{1}{2}\left[\frac{(x-0)}{1}\right]^2}$$



from $[-3, 1.3]$

Or, we can use the z Tables to find $P[Z < 1.3]$

Calculating Normal Dist. Probab.: Example 2, Contd.

Table 2: Cumulative standard normal probabilities: $P[Z \leq z]$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633

Using the table z Table above, we find $P[Z < 1.3] =$

Calculating Normal Dist. Probab.: Example 2, Contd.

Table 2: Cumulative standard normal probabilities: $P[Z \leq z]$

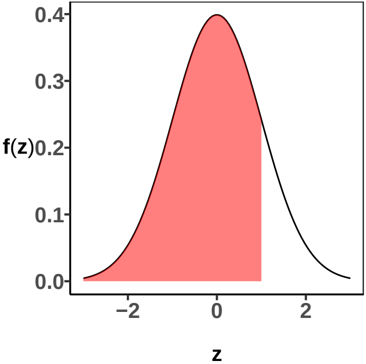
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633

Using the table z Table above, we find $P[Z < 1.3] = 0.9032$

Calculating Normal Dist. Probab.: Example 3

Suppose $X \sim \mathcal{N}(10, 3)$, what is $P[X < 13]$?

$$P[X < 13] = P[\frac{X-\mu}{\sigma} < \frac{13-\mu}{\sigma}]$$
$$= P[Z < \frac{13-10}{3}]$$
$$= P[Z < 1]$$



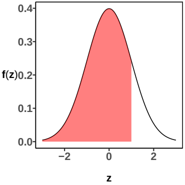
We can find $P[Z < 1]$ in the z table.

Calculating Normal Dist. Probab.: Example 3, Contd.

What is $P[Z < 1]$?

Table 3: Cumulative standard normal probabilities: $P[Z \leq z]$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177



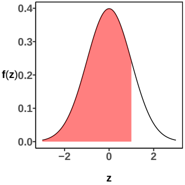
The chart to the left shows the area under the curve to the left of $Z = 1$.
From the z Table, $P[Z < 1] =$

Calculating Normal Dist. Probab.: Example 3, Contd.

What is $P[Z < 1]$?

Table 3: Cumulative standard normal probabilities: $P[Z \leq z]$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177



The chart to the left shows the area under the curve to the left of $Z = 1$.
From the z Table, $P[Z < 1] = 0.8413$

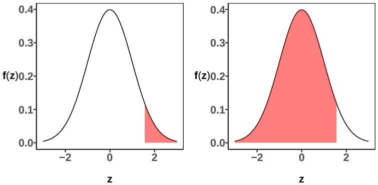
Finding probabilities in the z Table: Example 1

What is $P[Z > 1.56]$?

Table 4: Cumulative standard normal probabilities: $P[Z \leq z]$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545

$$P[Z > 1.56] = 1 - P[Z < 1.56]$$



Finding probabilities in the z Table: Example 1

What is $P[Z > 1.56]$?

Table 4: Cumulative standard normal probabilities: $P[Z \leq z]$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
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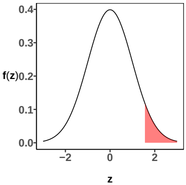
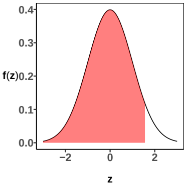
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545

$$
\begin{aligned}
P[Z > 1.56] &= 1 - P[Z < 1.56] \\
&= 1 - 0.9406
\end{aligned}
$$





Finding probabilities in the z Table: Example 1

What is $P[Z > 1.56]$?

Table 4: Cumulative standard normal probabilities: $P[Z \leq z]$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
1	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
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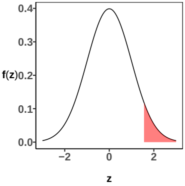
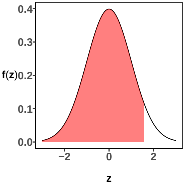
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545

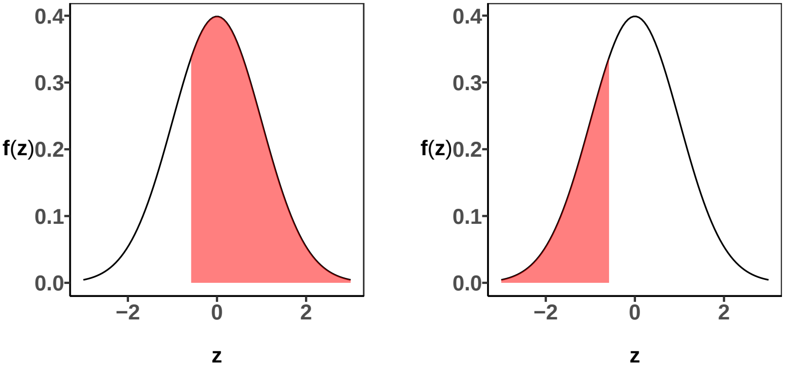
$$
\begin{aligned}
P[Z > 1.56] &= 1 - P[Z < 1.56] \\
&= 1 - 0.9406 \\
&= 0.0594
\end{aligned}
$$





Finding probabilities in the z Table: Example 2

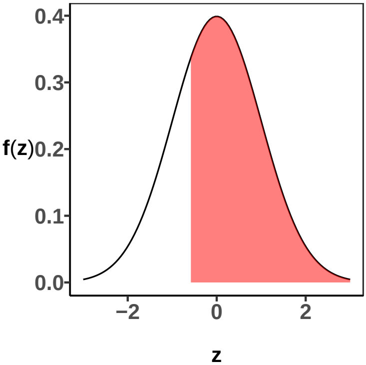
What is $P[Z > -0.58]$?

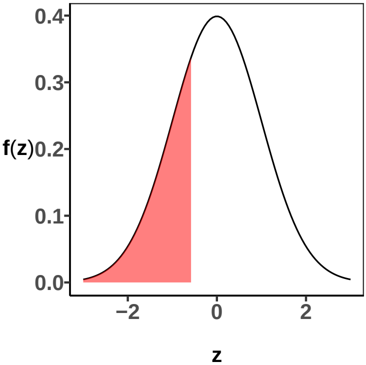


$$P[Z > -0.58] = 1 - P[Z < -0.58]$$

Finding probabilities in the z Table: Example 2

What is $P[Z > -0.58]$?

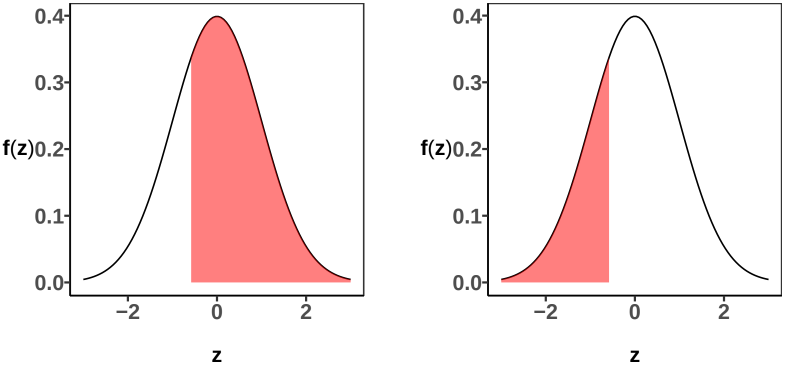




$$
\begin{aligned}
P[Z > -0.58] &= 1 - P[Z < -0.58] \\
&= 1 - 0.281
\end{aligned}
$$

Finding probabilities in the z Table: Example 2

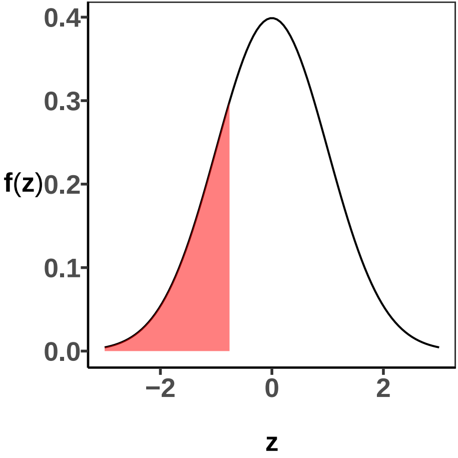
What is $P[Z > -0.58]$?



$$
\begin{aligned}
P[Z > -0.58] &= 1 - P[Z < -0.58] \\
&= 1 - 0.281 \\
&= 0.719
\end{aligned}
$$

Finding probabilities in the z Table: Example 3

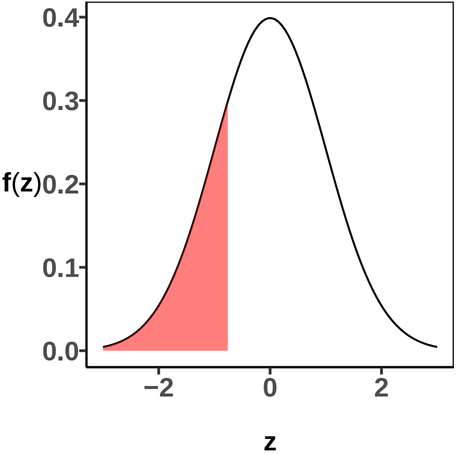
What is $P[Z < -0.76]$?



From the Z table, we can directly read $P[Z < -0.76] =$

Finding probabilities in the z Table: Example 3

What is $P[Z < -0.76]$?



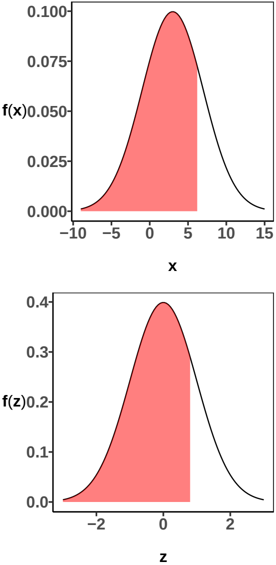
From the Z table, we can directly read $P[Z < -0.76] = 0.2236$

Calculating Prob. from the Normal Dist.

Suppose $X \sim \mathcal{N}(3, 4)$, what is $P[X < 6.2]$? Recall that we first need to standardize $P[X < 6.2]$.

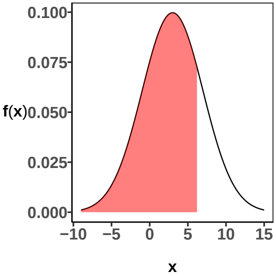
The figure in the upper right corner shows the area under the curve before standardization.

$$P[X < 6.2] = P[\frac{X - \mu}{\sigma} < \frac{6.2 - \mu}{\sigma}]$$



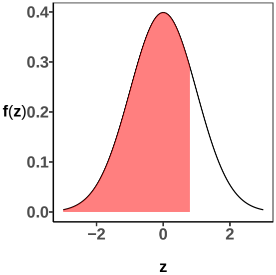
Calculating Prob. from the Normal Dist.

Suppose $X \sim \mathcal{N}(3,4)$, what is $P[X < 6.2]$? Recall that we first need to standardize $P[X < 6.2]$.



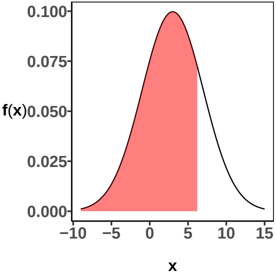
The figure in the upper right corner shows the area under the curve before standardization.

$$P[X < 6.2] = P[\frac{X-\mu}{\sigma} < \frac{6.2-\mu}{\sigma}]$$
$$= P[Z < \frac{6.2-3}{4}]$$



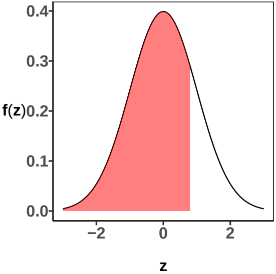
Calculating Prob. from the Normal Dist.

Suppose $X \sim \mathcal{N}(3,4)$, what is $P[X < 6.2]$? Recall that we first need to standardize $P[X < 6.2]$.



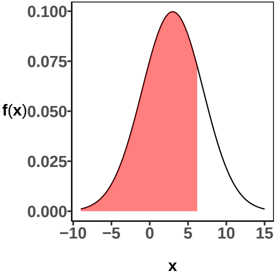
The figure in the upper right corner shows the area under the curve before standardization.

$$P[X < 6.2] = P[\frac{X-\mu}{\sigma} < \frac{6.2-\mu}{\sigma}]$$
$$= P[Z < \frac{6.2-3}{4}]$$
$$= P[Z < 0.8]$$



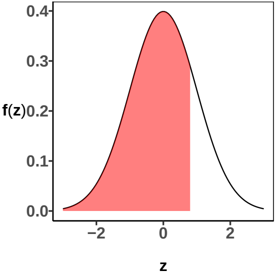
Calculating Prob. from the Normal Dist.

Suppose $X \sim \mathcal{N}(3, 4)$, what is $P[X < 6.2]$? Recall that we first need to standardize $P[X < 6.2]$.



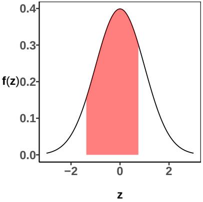
The figure in the upper right corner shows the area under the curve before standardization.

$$P[X < 6.2] = P[\frac{X - \mu}{\sigma} < \frac{6.2 - \mu}{\sigma}]$$
$$= P[Z < \frac{6.2 - 3}{4}]$$
$$= P[Z < 0.8]$$
$$= 0.7881$$



Calculating Normal Dist. Probab.: Example 4

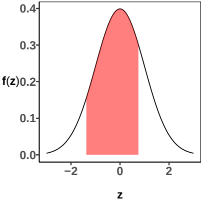
Suppose $X \sim \mathcal{N}(3, 4)$, what is $P[-2.5 < X < 6]$?



$$P[-2.5 < X < 6] = P[\frac{-2.5 - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{6 - \mu}{\sigma}]$$

Calculating Normal Dist. Probab.: Example 4

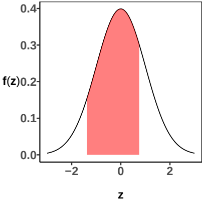
Suppose $X \sim \mathcal{N}(3, 4)$, what is $P[-2.5 < X < 6]$?



$$
\begin{aligned}
P[-2.5 < X < 6] &= P[\frac{-2.5-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{6-\mu}{\sigma}] \\
&= P[\frac{-2.5-3}{4} < Z < \frac{6-3}{4}]
\end{aligned}
$$

Calculating Normal Dist. Probab.: Example 4

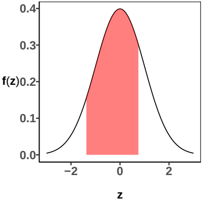
Suppose $X \sim \mathcal{N}(3, 4)$, what is $P[-2.5 < X < 6]$?



$$P[-2.5 < X < 6] = P[\frac{-2.5 - \mu}{\sigma} < \frac{X - \mu}{\sigma} < \frac{6 - \mu}{\sigma}]$$
$$= P[\frac{-2.5 - 3}{4} < Z < \frac{6 - 3}{4}]$$
$$= P[-1.38 < Z < 0.75]$$

Calculating Normal Dist. Probab.: Example 4

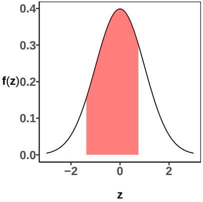
Suppose $X \sim \mathcal{N}(3,4)$, what is $P[-2.5 < X < 6]$?



$$P[-2.5 < X < 6] = P[\frac{-2.5-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{6-\mu}{\sigma}]$$
$$= P[\frac{-2.5-3}{4} < Z < \frac{6-3}{4}]$$
$$= P[-1.38 < Z < 0.75]$$
$$= P[Z < 0.75] - P[Z < -1.38]$$

Calculating Normal Dist. Probab.: Example 4

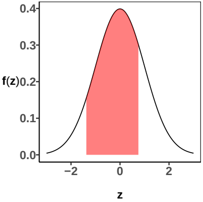
Suppose $X \sim \mathcal{N}(3, 4)$, what is $P[-2.5 < X < 6]$?



$$
\begin{aligned}
P[-2.5 < X < 6] &= P[\frac{-2.5-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{6-\mu}{\sigma}] \\
&= P[\frac{-2.5-3}{4} < Z < \frac{6-3}{4}] \\
&= P[-1.38 < Z < 0.75] \\
&= P[Z < 0.75] - P[Z < -1.38] \\
&= 0.7734 - 0.0838
\end{aligned}
$$

Calculating Normal Dist. Probab.: Example 4

Suppose $X \sim \mathcal{N}(3, 4)$, what is $P[-2.5 < X < 6]$?



$$
\begin{aligned}
P[-2.5 < X < 6] &= P[\frac{-2.5-\mu}{\sigma} < \frac{X-\mu}{\sigma} < \frac{6-\mu}{\sigma}] \\
&= P[\frac{-2.5-3}{4} < Z < \frac{6-3}{4}] \\
&= P[-1.38 < Z < 0.75] \\
&= P[Z < 0.75] - P[Z < -1.38] \\
&= 0.7734 - 0.0838 \\
&= 0.6896
\end{aligned}
$$

The Normal Distribution: Problem 1

Suppose babies weight at birth, X, is $\mathcal{N}(3500, 500)$. What is the probability that a baby is born at a weight: Less than 3100g? More than 4000g? Between 2000g and 4000g?

Answer:
$P[X < 3100] =$

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Answer:
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Answer:

$P[X < 3100] = P[\frac{X-3500}{500} < \frac{3100-3500}{500}] = P[Z < -0.8] = 0.2119$

$P[X > 4000] =$

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$$P[2000 < X < 4000] = P[\frac{2000-3500}{500} < \frac{X-3500}{500} < \frac{4000-3500}{500}]$$

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$$\begin{aligned} P[2000 < X < 4000] &= P[\frac{2000-3500}{500} < \frac{X-3500}{500} < \frac{4000-3500}{500}] \\ &= P[-3 < Z < 1] \end{aligned}$$

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$$\begin{aligned} P[2000 < X < 4000] &= P[\frac{2000-3500}{500} < \frac{X-3500}{500} < \frac{4000-3500}{500}] \\ &= P[-3 < Z < 1] \\ &= P[Z < 1] - P[Z < -3] \end{aligned}$$

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$$\begin{aligned} P[2000 < X < 4000] &= P[\frac{2000-3500}{500} < \frac{X-3500}{500} < \frac{4000-3500}{500}] \\ &= P[-3 < Z < 1] \\ &= P[Z < 1] - P[Z < -3] \\ &= 0.8413 - 0.0013 \\ &= 0.84 \end{aligned}$$

The Normal Distribution: Problem 2

Scores on the GMAT are roughly normally distributed with a mean of 527 and a standard deviation of 112. What is the probability of an individual scoring above 500 on the GMAT?

Answer:
$X \sim \mathcal{N}(527, 112)$.

$$P[X > 500] =$$

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Answer:
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$$P[X > 500] = 1 - P[X < 500] =$$

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$$P[X > 500] = 1 - P[X < 500] = 1 - P[Z < -0.24] =$$

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$$P[X > 500] = 1 - P[X < 500] = 1 - P[Z < -0.24] = 1 - 0.4052 =$$

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Answer:
$X \sim \mathcal{N}(527, 112)$.

$$P[X > 500] = 1 - P[X < 500] = 1 - P[Z < -0.24] = 1 - 0.4052 = 0.5948$$

The Normal Distribution: Problem 3

The length of human pregnancies from conception to birth approximates a normal distribution with a mean of 266 days and a standard deviation of 16 days. What proportion of all pregnancies will last between 240 and 270 days (roughly between 8 and 9 months)?

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$$P[240 < X < 270] = P[-1.62 < Z < 0.25]$$

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$$\begin{aligned} P[240 < X < 270] &= P[-1.62 < Z < 0.25] \\ &= P[Z < 0.25] - P[Z < -1.62] \end{aligned}$$

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$$\begin{aligned} P[240 < X < 270] &= P[-1.62 < Z < 0.25] \\ &= P[Z < 0.25] - P[Z < -1.62] \\ &= 0.5987 - 0.0526 \end{aligned}$$

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$$
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P[240 < X < 270] &= P[-1.62 < Z < 0.25] \\
&= P[Z < 0.25] - P[Z < -1.62] \\
&= 0.5987 - 0.0526 \\
&= 0.5461
\end{aligned}
$$

The Normal Distribution: Problem 4

The average number of acres burned by forest and range fires in a large New Mexico county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What is the probability that between 2,500 and 4,200 acres will be burned in any given yea?

The Normal Distribution: Problem 4

The average number of acres burned by forest and range fires in a large New Mexico county is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What is the probability that between 2,500 and 4,200 acres will be burned in any given yea?

Answer:
$X \sim \mathcal{N}(4300, 750)$.

$$P[2500 < X < 4200] = P[-2.4 < Z < -0.13]$$

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$$\begin{aligned} P[2500 < X < 4200] &= P[-2.4 < Z < -0.13] \\ &= P[Z < -0.13] - P[Z < -2.4] \\ &= 0.4483 - 0.0082 \end{aligned}$$

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The Normal Distribution: Problem 5

A theater chain has studied its movie customers to determine how much money they spend on concessions. The study revealed that the spending distribution is approximately normally distributed with a mean of \$4.11 and a standard deviation of \$1.37. What percentage of customers will spend less than \$3 on concessions?

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Answer:
$X \sim \mathcal{N}(4.11, 1.37)$.

$$P[X < 3] =$$

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$$P[X < 3] = P[Z < -0.81] =$$

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Answer:
$X \sim \mathcal{N}(4.11, 1.37)$.

$$P[X < 3] = P[Z < -0.81] = 0.209$$

The Normal Distribution: Problem 6

A baker knows that the daily demand, X, for apple pies is normally distributed with a mean of 50 pies and standard deviation of 5 pies. Determine $P[X = 40]$, $P[X < 38]$, $P[X > 58]$, $P[36 < X < 46]$, $P[56 < X < 62]$, and $P[47 < X < 54]$.

The Normal Distribution: Problem 6

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Answer:
$X \sim \mathcal{N}(50, 5)$.
$P[X = 40] =$

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Answer:
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$P[X = 40] = 0$
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$P[56 < X < 62] = 0.1069$
$P[47 < X < 54] = 0.5139$

Using the Normal Dist. tables backwards

The grades of 500 students in an exam are normally distributed with a mean of 45 and a standard deviation of 20. If 20% of candidates made it to the dean's list by scoring x or more, estimate the value of x.

We know that $X \sim \mathcal{N}(45, 20)$, so $P[X > x] = 0.2$, which implies that $P[X < x] = 0.8$, which means:

$P[X < x] = 0.8 \Rightarrow P[\frac{X-\mu}{\sigma} < \frac{x-\mu}{\sigma}] = 0.8 \Rightarrow P[Z < \frac{x-45}{20}] = 0.8$

Now we need to find a z score that is associated with the probability 0.8. From the z table we can find this to be $Z = 0.84$. We can use this z to solve for x:

$$
\begin{aligned}
P[Z < \frac{x-45}{20}] = 0.8 &\Rightarrow z = 0.84 \\
&\Rightarrow 0.84 = \frac{x-45}{20} \\
&\Rightarrow x = 20 * 0.84 + 45 \\
&\Rightarrow x = 61.8
\end{aligned}
$$

So, students scoring more than 61.8 made the dean's list.

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Answer:
$X \sim \mathcal{N}(527, 122)$.
We need to solve for x such that $P[X \geq x] = 0.05$

$$P[Z > \frac{x - 527}{122}] = 0.05$$

In order to score in the highest 5%, you need more than 728.3.

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$$P[Z > \frac{x-527}{122}] = 0.05$$
$$\Rightarrow P[Z < \frac{x-527}{122}] = 1 - 0.05 = 0.95$$

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$$\begin{aligned} P[Z > \frac{x-527}{122}] &= 0.05 \\ &\Rightarrow P[Z < \frac{x-527}{122}] = 1 - 0.05 = 0.95 \\ &\Rightarrow z = 1.65 \end{aligned}$$

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$$\begin{aligned} P[Z > \frac{x-527}{122}] = 0.05 & \\ & \Rightarrow P[Z < \frac{x-527}{122}] = 1 - 0.05 = 0.95 \\ & \Rightarrow z = 1.65 \\ & \Rightarrow \frac{x-527}{122} = 1.65 \\ & \Rightarrow x = 728.3 \end{aligned}$$

In order to score in the highest 5%, you need more than 728.3.

Using the Z tables backwards: Problem 2

The length of human pregnancies from conception to birth approximates a normal distribution with a mean of 266 days and a standard deviation of 16 days. What length of time marks the shortest 70% of all pregnancies?

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Answer:
$X \sim \mathcal{N}(266, 16)$.
We want to solve for x such that $P[X < x] = 0.7$

$$P[Z < \frac{x - 266}{16}] = 0.7$$

70% of pregnancies take place before 274.32 days.

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$$P[Z < \frac{x - 266}{16}] = 0.7$$
$$\Rightarrow z = 0.52$$

70% of pregnancies take place before 274.32 days.

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$$\begin{aligned} P[Z < \frac{x-266}{16}] &= 0.7 \\ &\Rightarrow z = 0.52 \\ &\Rightarrow \frac{x-266}{16} = 0.52 \end{aligned}$$

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$$\begin{aligned} P[Z < \frac{x-266}{16}] &= 0.7 \\ &\Rightarrow z = 0.52 \\ &\Rightarrow \frac{x-266}{16} = 0.52 \\ &\Rightarrow x = 274.32 \end{aligned}$$

70% of pregnancies take place before 274.32 days.

Using the Z tables backwards: Problem 3

The average number of acres burned by forest and range fires is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What number of burnt acres corresponds to the 38^{th} percentile?

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Answer:
$X \sim \mathcal{N}(4300, 750)$.
We want to solve for x such that $P[X < x] = 0.38$

$$P[Z < \frac{x - 4300}{750}] = 0.38$$

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Answer:
$X \sim \mathcal{N}(4300, 750)$.
We want to solve for x such that $P[X < x] = 0.38$

$$P[Z < \frac{x - 4300}{750}] = 0.38$$

$$\Rightarrow z = -0.31$$

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$$P[Z < \frac{x - 4300}{750}] = 0.38$$

$$\Rightarrow z = -0.31$$

$$\Rightarrow \frac{x - 4300}{750} = -0.31$$

Using the Z tables backwards: Problem 3

The average number of acres burned by forest and range fires is 4,300 acres per year, with a standard deviation of 750 acres. The distribution of the number of acres burned is normal. What number of burnt acres corresponds to the 38^{th} percentile?

Answer:
$X \sim \mathcal{N}(4300, 750)$.
We want to solve for x such that $P[X < x] = 0.38$

$$P[Z < \frac{x - 4300}{750}] = 0.38$$

$$\Rightarrow z = -0.31$$

$$\Rightarrow \frac{x - 4300}{750} = -0.31$$

$$\Rightarrow x = 4067.5$$

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$$P[Z < \frac{x - 4300}{750}] = 0.38$$
$$\Rightarrow z = -0.31$$
$$\Rightarrow \frac{x - 4300}{750} = -0.31$$
$$\Rightarrow x = 4067.5$$

The bottom 38% of fires burned up to 4067.5 acres of land.

Using the Z tables backwards: Problem 4

A theater chain has studied its movie customers to determine how much money they spend on concessions. The study revealed that the spending distribution is approximately normally distributed with a mean of \$4.11 and a standard deviation of \$1.37. What spending amount corresponds to the top 87^{th} percentile?

Using the Z tables backwards: Problem 4

A theater chain has studied its movie customers to determine how much money they spend on concessions. The study revealed that the spending distribution is approximately normally distributed with a mean of \$4.11 and a standard deviation of \$1.37. What spending amount corresponds to the top 87^{th} percentile?

Answer:
$X \sim \mathcal{N}(4.11, 1.37)$.
We need to solve for x such that $P[X \geq x] = 0.87$

$$P[Z > \frac{x - 4.11}{1.37}] = 0.87 \Rightarrow P[Z < \frac{x - 4.11}{1.37}] = 1 - 0.87 = 0.13$$

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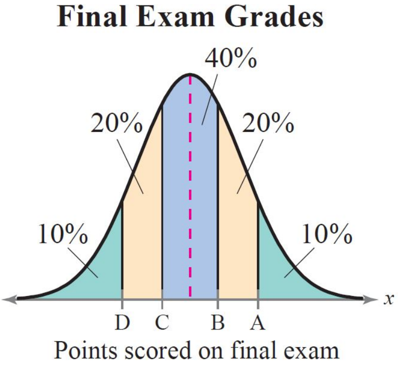
The top 87% spend more than 2.5619 on concessions.

Using the Z tables backwards: Problem 5

Suppose your final grades are normally distributed with a mean of 72 and a standard deviation of 9. Also suppose I assigned you letter grades according to the following rule: the top 10% receive As, the next 20% receive Bs, the middle 40% receive Cs, the next 20% receive Ds, and the bottom 10% receive Fs. Find the lowest grade that would qualify a student for an A, a B, a C, and a D.

Using the Z tables backwards: Problem 5

Suppose your final grades are normally distributed with a mean of 72 and a standard deviation of 9. Also suppose I assigned you letter grades according to the following rule: the top 10% receive As, the next 20% receive Bs, the middle 40% receive Cs, the next 20% receive Ds, and the bottom 10% receive Fs. Find the lowest grade that would qualify a student for an A, a B, a C, and a D.



Using the Z tables backwards: Prblm 5, Contd.

Answer:
$X \sim \mathcal{N}(72, 9)$.
For A: We need to solve for x such that $P[X \geq x] = 0.1$

$$P[Z > \frac{x-72}{9}] = 0.1$$

Using the Z tables backwards: Prblm 5, Contd.

Answer:
$X \sim \mathcal{N}(72, 9)$.
For A: We need to solve for x such that $P[X \geq x] = 0.1$

$$P[Z > \frac{x - 72}{9}] = 0.1$$
$$\Rightarrow P[Z < \frac{x - 72}{9}] = 1 - 0.1 = 0.9$$

Using the Z tables backwards: Prblm 5, Contd.

Answer:
$X \sim \mathcal{N}(72, 9)$.
For A: We need to solve for x such that $P[X \geq x] = 0.1$

$$
\begin{aligned}
P[Z > \frac{x - 72}{9}] &= 0.1 \\
&\Rightarrow P[Z < \frac{x - 72}{9}] = 1 - 0.1 = 0.9 \\
&\Rightarrow z = 1.28
\end{aligned}
$$

Answer:
$X \sim \mathcal{N}(72, 9)$.
For A: We need to solve for x such that $P[X \geq x] = 0.1$

$$\begin{aligned} P[Z > \frac{x-72}{9}] &= 0.1 \\ &\Rightarrow P[Z < \frac{x-72}{9}] = 1 - 0.1 = 0.9 \\ &\Rightarrow z = 1.28 \\ &\Rightarrow \frac{x-72}{9} = 1.28 \end{aligned}$$

Using the Z tables backwards: Prblm 5, Contd.

Answer:
$X \sim \mathcal{N}(72, 9)$.
For A: We need to solve for x such that $P[X \geq x] = 0.1$

$$
\begin{aligned}
P[Z > \frac{x-72}{9}] = 0.1 & \\
& \Rightarrow P[Z < \frac{x-72}{9}] = 1 - 0.1 = 0.9 \\
& \Rightarrow z = 1.28 \\
& \Rightarrow \frac{x-72}{9} = 1.28 \\
& \Rightarrow x = 83.52
\end{aligned}
$$

Answer:
$X \sim \mathcal{N}(72, 9)$.
For A: We need to solve for x such that $P[X \geq x] = 0.1$

$$\begin{aligned} P[Z > \frac{x-72}{9}] &= 0.1 \\ &\Rightarrow P[Z < \frac{x-72}{9}] = 1 - 0.1 = 0.9 \\ &\Rightarrow z = 1.28 \\ &\Rightarrow \frac{x-72}{9} = 1.28 \\ &\Rightarrow x = 83.52 \end{aligned}$$

So, you'll get an A if you score [83.52 , 100]

Answer:
$X \sim \mathcal{N}(72, 9)$.
For B: We need to solve for x such that $P[X < x] = 0.7$

$$P[Z < \frac{x - 72}{9}] = 0.7$$

Using the Z tables backwards: Prblm 5, Contd.

Answer:
$X \sim \mathcal{N}(72, 9)$.
For B: We need to solve for x such that $P[X < x] = 0.7$

$$P[Z < \frac{x - 72}{9}] = 0.7$$
$$\Rightarrow z = 0.52$$

Using the Z tables backwards: Prblm 5, Contd.

Answer:
$X \sim \mathcal{N}(72, 9)$.
For B: We need to solve for x such that $P[X < x] = 0.7$

$$\begin{aligned} P[Z < \frac{x-72}{9}] &= 0.7 \\ &\Rightarrow z = 0.52 \\ &\Rightarrow \frac{x-72}{9} = 0.52 \end{aligned}$$

Answer:
$X \sim \mathcal{N}(72, 9)$.
For B: We need to solve for x such that $P[X < x] = 0.7$

$$P[Z < \frac{x-72}{9}] = 0.7$$
$$\Rightarrow z = 0.52$$
$$\Rightarrow \frac{x-72}{9} = 0.52$$
$$\Rightarrow x = 76.68$$

Answer:
$X \sim \mathcal{N}(72, 9)$.
For B: We need to solve for x such that $P[X < x] = 0.7$

$$\begin{aligned} P[Z < \frac{x-72}{9}] &= 0.7 \\ &\Rightarrow z = 0.52 \\ &\Rightarrow \frac{x-72}{9} = 0.52 \\ &\Rightarrow x = 76.68 \end{aligned}$$

Using the Z tables backwards: Prblm 5, Contd.

Answer:
$X \sim \mathcal{N}(72, 9)$.
For B: We need to solve for x such that $P[X < x] = 0.7$

$$P[Z < \frac{x-72}{9}] = 0.7$$
$$\Rightarrow z = 0.52$$
$$\Rightarrow \frac{x-72}{9} = 0.52$$
$$\Rightarrow x = 76.68$$

So, you'll get a B if you score [76.68 , 83.52)
You can similarly find the cutoff for C, D, and F.

Finding the Standard Deviation: Example

The number of daily car sales X at a dealer is normally distributed with mean $\mu = 2$. If $P[X > 3] = 0.1587$, what is σ_X?
$X \sim \mathcal{N}(2, \sigma_X)$.
We need to solve for σ_X such that $P[\frac{X-\mu}{\sigma_X} > \frac{3-2}{\sigma_X}] = P[Z > \frac{3-2}{\sigma_X}] = 0.1587$

$$P[Z > \frac{3-2}{\sigma_X}] = 0.1587 \Rightarrow P[Z < \frac{1}{\sigma_X}] = 1 - 0.1587 = 0.8413$$

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$$P[Z > \frac{3-2}{\sigma_X}] = 0.1587 \Rightarrow P[Z < \frac{1}{\sigma_X}] = 1 - 0.1587 = 0.8413$$
$$\Rightarrow z = 1$$

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&\Rightarrow \frac{1}{\sigma_X} = 1
\end{aligned}
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\Rightarrow z = 1 \\
\Rightarrow \frac{1}{\sigma_X} = 1 \\
\Rightarrow \sigma_X = 1
\end{aligned}
$$

Finding the Standard Deviation: Example

The number of daily car sales X at a dealer is normally distributed with mean $\mu = 2$. If $P[X > 3] = 0.1587$, what is σ_X?
$X \sim \mathcal{N}(2, \sigma_X)$.
We need to solve for σ_X such that $P[\frac{X-\mu}{\sigma_X} > \frac{3-2}{\sigma_X}] = P[Z > \frac{3-2}{\sigma_X}] = 0.1587$

$$
\begin{aligned}
P[Z > \frac{3-2}{\sigma_X}] = 0.1587 &\Rightarrow P[Z < \frac{1}{\sigma_X}] = 1 - 0.1587 = 0.8413 \\
&\Rightarrow z = 1 \\
&\Rightarrow \frac{1}{\sigma_X} = 1 \\
&\Rightarrow \sigma_X = 1
\end{aligned}
$$

So, the number of daily car sales is $\mathcal{N}(2, 1)$.

Finding the Standard Deviation: Problem

The average freshman student spends \$1200 on textbooks. Let X represent student spending. If 0.13% of freshmen students manage to spend less than \$600 on textbooks, what is σ_X?

Answer:

Finding the Standard Deviation: Problem

The average freshman student spends \$1200 on textbooks. Let X represent student spending. If 0.13% of freshmen students manage to spend less than \$600 on textbooks, what is σ_X?

Answer:

$X \sim \mathcal{N}(1200, \sigma_X)$. We know that $P[X < 600] = 0.0013$.

We need to solve for σ_X such that

$P[\frac{X-\mu}{\sigma_X} < \frac{600-1200}{\sigma_X}] = P[Z < \frac{-600}{\sigma_X}] = 0.0013$

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$$P[Z < \frac{-600}{\sigma_X}] = 0.0013$$

$$\Rightarrow z = -3$$

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$$\begin{aligned} P[Z < \frac{-600}{\sigma_X}] &= 0.0013 \\ &\Rightarrow z = -3 \\ &\Rightarrow \frac{-600}{\sigma_X} = -3 \end{aligned}$$

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$P[\frac{X-\mu}{\sigma_X} < \frac{600-1200}{\sigma_X}] = P[Z < \frac{-600}{\sigma_X}] = 0.0013$

$$\begin{aligned} P[Z < \frac{-600}{\sigma_X}] &= 0.0013 \\ &\Rightarrow z = -3 \\ &\Rightarrow \frac{-600}{\sigma_X} = -3 \\ &\Rightarrow \sigma_X = \frac{-600}{-3} \\ &\Rightarrow \sigma_X = 200 \end{aligned}$$

Finding the Mean: Problem

Suppose that the weight X in pounds, of a 40 year old man is a normally distributed with a standard deviation $\sigma_X = 20$ pounds. If 5% of this population is heavier than 214 pounds, what is the mean μ_X of this distribution?

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Answer:
$X \sim \mathcal{N}(\mu_X, 20)$. We know that $P[X > 214] = 0.05$.
We need to solve for μ_X such that
$P[\frac{X-\mu_X}{\sigma_X} > \frac{214-\mu_X}{20}] = P[Z > \frac{214-\mu_X}{20}] = 0.05$

$$P[Z > \frac{214 - \mu_X}{20}] = 0.05 \Rightarrow P[Z < \frac{214 - \mu_X}{20}] = 0.95$$

Finding the Mean: Problem

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$$P[Z > \frac{214 - \mu_X}{20}] = 0.05 \Rightarrow P[Z < \frac{214 - \mu_X}{20}] = 0.95$$
$$\Rightarrow z = 1.645$$

Finding the Mean: Problem

Suppose that the weight X in pounds, of a 40 year old man is a normally distributed with a standard deviation $\sigma_X = 20$ pounds. If 5% of this population is heavier than 214 pounds, what is the mean μ_X of this distribution?

Answer:

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We need to solve for μ_X such that

$P[\frac{X-\mu_X}{\sigma_X} > \frac{214-\mu_X}{20}] = P[Z > \frac{214-\mu_X}{20}] = 0.05$

$$
\begin{aligned}
P[Z > \frac{214-\mu_X}{20}] = 0.05 &\Rightarrow P[Z < \frac{214-\mu_X}{20}] = 0.95 \\
&\Rightarrow z = 1.645 \\
&\Rightarrow \frac{214-\mu_X}{20} = 1.645
\end{aligned}
$$

Finding the Mean: Problem

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Answer:
$X \sim \mathcal{N}(\mu_X, 20)$. We know that $P[X > 214] = 0.05$.
We need to solve for μ_X such that
$P[\frac{X-\mu_X}{\sigma_X} > \frac{214-\mu_X}{20}] = P[Z > \frac{214-\mu_X}{20}] = 0.05$

$$\begin{aligned} P[Z > \frac{214-\mu_X}{20}] = 0.05 &\Rightarrow P[Z < \frac{214-\mu_X}{20}] = 0.95 \\ &\Rightarrow z = 1.645 \\ &\Rightarrow \frac{214-\mu_X}{20} = 1.645 \\ &\Rightarrow \mu_X = 214 - 20 * 1.645 \\ &\Rightarrow \mu_X = 181.1 \end{aligned}$$

Finding the Mean: Problem

The breaking distance X in feet is a normally distributed with a standard deviation $\sigma_X = 10$. If 0.62% of cars achieve a breaking distance of 225 feet or less, what is the mean μ_X of this distribution?

Finding the Mean: Problem

The breaking distance X in feet is a normally distributed with a standard deviation $\sigma_X = 10$. If 0.62% of cars achieve a breaking distance of 225 feet or less, what is the mean μ_X of this distribution?

Answer:

$X \sim \mathcal{N}(\mu_X, 10)$. We know that $P[X < 225] = 0.0062$.
We need to solve for μ_X such that $P[\frac{X-\mu_X}{\sigma_X} < \frac{225-\mu_X}{10}] = 0.0062$

$$P[Z < \frac{225 - \mu_X}{10}] = 0.0062$$

Finding the Mean: Problem

The breaking distance X in feet is a normally distributed with a standard deviation $\sigma_X = 10$. If 0.62% of cars achieve a breaking distance of 225 feet or less, what is the mean μ_X of this distribution?

Answer:

$X \sim \mathcal{N}(\mu_X, 10)$. We know that $P[X < 225] = 0.0062$.
We need to solve for μ_X such that $P[\frac{X-\mu_X}{\sigma_X} < \frac{225-\mu_X}{10}] = 0.0062$

$$P[Z < \frac{225 - \mu_X}{10}] = 0.0062$$

$$\Rightarrow z = -2.5$$

Finding the Mean: Problem

The breaking distance X in feet is a normally distributed with a standard deviation $\sigma_X = 10$. If 0.62% of cars achieve a breaking distance of 225 feet or less, what is the mean μ_X of this distribution?

Answer:

$X \sim \mathcal{N}(\mu_X, 10)$. We know that $P[X < 225] = 0.0062$.
We need to solve for μ_X such that $P[\frac{X-\mu_X}{\sigma_X} < \frac{225-\mu_X}{10}] = 0.0062$

$$
\begin{aligned}
P[Z < \frac{225-\mu_X}{10}] &= 0.0062 \\
&\Rightarrow z = -2.5 \\
&\Rightarrow \frac{225-\mu_X}{10} = -2.5
\end{aligned}
$$

Finding the Mean: Problem

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We need to solve for μ_X such that $P[\frac{X-\mu_X}{\sigma_X} < \frac{225-\mu_X}{10}] = 0.0062$

$$\begin{aligned} P[Z < \frac{225-\mu_X}{10}] &= 0.0062 \\ &\Rightarrow z = -2.5 \\ &\Rightarrow \frac{225-\mu_X}{10} = -2.5 \\ &\Rightarrow \mu_X = 225 + 10 * 2.5 \\ &\Rightarrow \mu_X = 250 \end{aligned}$$