Chapter 6: Discrete Probability Distributions

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Introduction

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$$X = \{1, 2, 3, 4, 5, 6, ...\}$$

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Answer: $P[X = 1] = 0.5$

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Random Variables

Definition

A ***random variable*** is a variable that takes on different values according to the outcome of an experiment.

Discrete Random Variable: Only takes on certain values along an interval.

Continuous Random Variable: Takes on any value within an interval

Example

Suppose we have a class of 24 students.

- ***Discrete Random Variable***: The number of students eligible to vote this year.
- ***Continuous Random Variable***: The height of a student in this class.

Probability Distribution

Definitions

We can define a ***probability distribution*** as the relative frequency distribution that should theoretically occur for observations from a given population.

Probability Distribution, Example 1

Let X represent the number of times it takes before a coin lands on heads. The probability distribution of X is

x	$P[X = x]$
1	
2	
3	
$\vdots$	$\vdots$
n	
$\vdots$	$\vdots$

Can you guess the average of X?

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$\vdots$	$\vdots$
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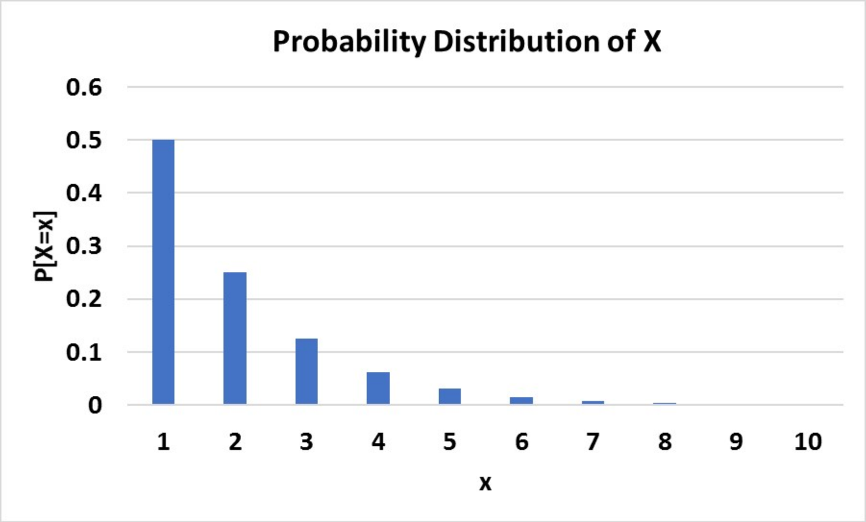
Probability Distribution, Example 1



Probability Distribution: Example 2

Suppose we roll two fair six-sided dice, let X represent their difference: 1^{st} Die -2^{nd} Die. The probability distribution of X is.

x	$P(x)$
-5	
-4	
-3	
-2	
-1	
0	
1	
2	
3	
4	
5	

$$P[X = 4] =$$
$$P[X = 0] =$$
$$P[0 < X < 3] =$$
$$P[0 \leq X \leq 3] =$$
$$P[X > -5] =$$
$$P[X < 3] =$$
$$P[-1 \geq X \geq -3] =$$

Think

What is $\bar{X}$ and s_X?

Probability Distribution: Example 2

Suppose we roll two fair six-sided dice, let X represent their difference: 1^{st} Die -2^{nd} Die. The probability distribution of X is.

x	$P(x)$
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-3	
-2	
-1	
0	
1	
2	
3	
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5	

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$$P[0 < X < 3] = P[X = 1] + P[X = 2] = \frac{9}{36}$$

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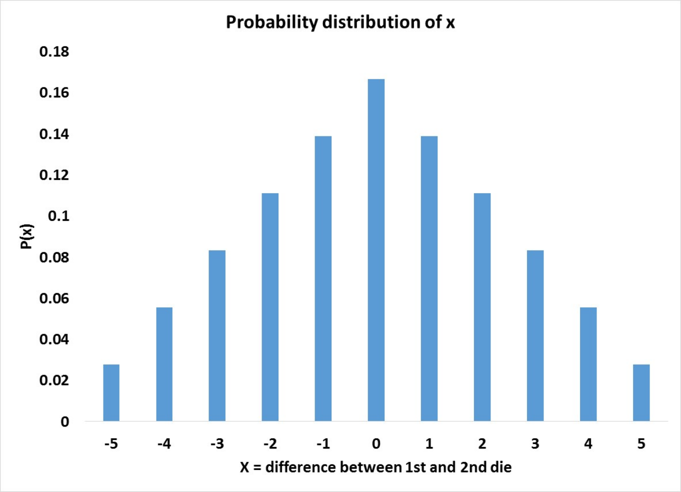
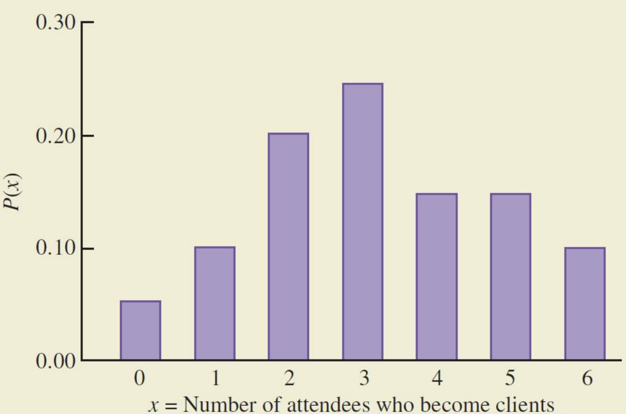


Discrete Prob. Distributions: characteristics

- $\forall x, 0 \leq P[X = x] \leq 1$
- $\sum_{i=1}^{n} P[X = x_i] = 1$
- The values of X are exhaustive: The probability distribution includes all possible values of X.
- The values of X are mutually exclusive: Only one value can occur for a given experiment.

Example 3: Using relative frequencies

A financial counselor conducts investment seminars to groups of 6 attendees, some of which become clients with the following relative frequency distribution.



Example 3, continued

In the previous example, note that the probability that:
All attendees become clients is $P[X = 6] = 0.1$,
None of them becomes a client is $P[X = 0] = 0.05$,
Half of them become clients is $P[X = 3] = 0.25$.

x	$P[x]$
0	
1	
2	
3	
4	
5	
6	

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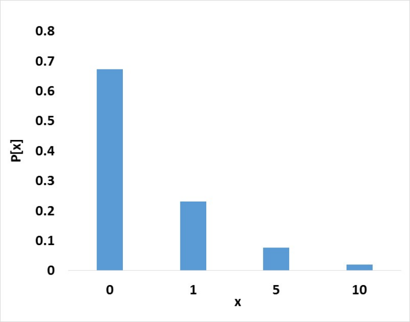
Discrete Prob. Dist.: Example 4

In a game of cards you win \$1 if you draw a spade, ♠, \$5 if you draw an ace (including the ace of spades), \$10 if you draw the king of clubs, ♣, and nothing for any other card you draw. Let X be a random variable representing the amount of money you can win.

x	$P[x]$
\$10	
\$5	
\$1	
\$0	



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x	$P[x]$
\$10	1/52
\$5	
\$1	
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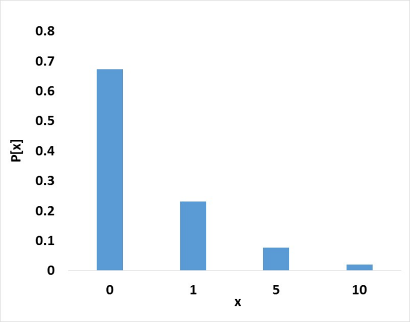

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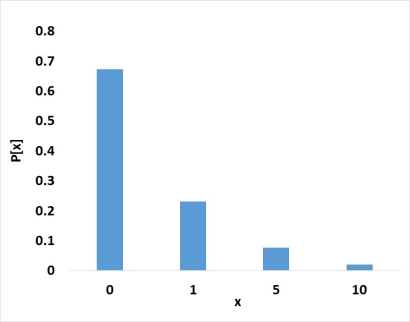
Discrete Prob. Dist.: Example 4

In a game of cards you win \$1 if you draw a spade, ♠, \$5 if you draw an ace (including the ace of spades), \$10 if you draw the king of clubs, ♣, and nothing for any other card you draw. Let X be a random variable representing the amount of money you can win.

x	$P[x]$
\$10	1/52
\$5	4/52
\$1	
\$0	



Think

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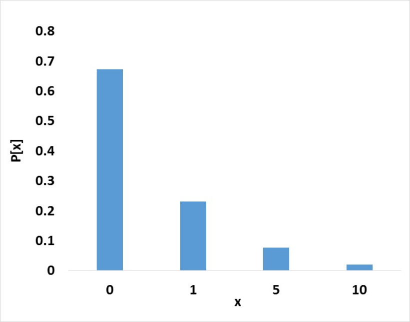
In a game of cards you win \$1 if you draw a spade, ♠, \$5 if you draw an ace (including the ace of spades), \$10 if you draw the king of clubs, ♣, and nothing for any other card you draw. Let X be a random variable representing the amount of money you can win.

x	$P[x]$
\$10	1/52
\$5	4/52
\$1	12/52
\$0	



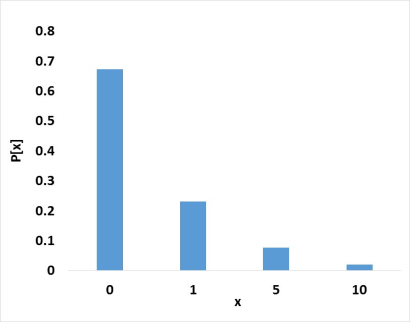
Think

What is $\bar{X}$ and s_X?

Discrete Prob. Dist.: Example 4

In a game of cards you win \$1 if you draw a spade, ♠, \$5 if you draw an ace (including the ace of spades), \$10 if you draw the king of clubs, ♣, and nothing for any other card you draw. Let X be a random variable representing the amount of money you can win.

x	$P[x]$
\$10	1/52
\$5	4/52
\$1	12/52
\$0	35/52



Think

What is $\bar{X}$ and s_X?

Reminder: The weighted average

Recall that we defined a weighted average as:

$$\bar{X}_w = \frac{\sum_{i=1}^{n} w_i \times x_i}{\sum_{i=1}^{n} w_i}$$

Where w_i is the weight corresponding to x_i.

The Mean of a Discrete Probability Distribution

Definition

The mean of a discrete probability distribution for a discrete random variable X is called the ***expected value***, $\mathbb{E}[X]$. It is a weighted average of all the possible outcomes, weighted according to their probability of occurrence.

$$\mu = \mathbb{E}[X] = \frac{\sum_{i=1}^{n} P[x_i] \times x_i}{\sum_{i=1}^{n} P[x_i]} = \sum_{i=1}^{n} P[x_i] \times x_i$$

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	
-4	0.056	
-3	0.083	
-2	0.111	
-1	0.139	
0	0.167	
1	0.139	
2	0.111	
3	0.083	
4	0.056	
5	0.028	

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	
-3	0.083	
-2	0.111	
-1	0.139	
0	0.167	
1	0.139	
2	0.111	
3	0.083	
4	0.056	
5	0.028	

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	
-2	0.111	
-1	0.139	
0	0.167	
1	0.139	
2	0.111	
3	0.083	
4	0.056	
5	0.028	

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	-0.250
-2	0.111	
-1	0.139	
0	0.167	
1	0.139	
2	0.111	
3	0.083	
4	0.056	
5	0.028	

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	-0.250
-2	0.111	-0.222
-1	0.139	
0	0.167	
1	0.139	
2	0.111	
3	0.083	
4	0.056	
5	0.028	

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	-0.250
-2	0.111	-0.222
-1	0.139	-0.139
0	0.167	
1	0.139	
2	0.111	
3	0.083	
4	0.056	
5	0.028	

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	-0.250
-2	0.111	-0.222
-1	0.139	-0.139
0	0.167	0
1	0.139	
2	0.111	
3	0.083	
4	0.056	
5	0.028	

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	-0.250
-2	0.111	-0.222
-1	0.139	-0.139
0	0.167	0
1	0.139	0.139
2	0.111	
3	0.083	
4	0.056	
5	0.028	

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	-0.250
-2	0.111	-0.222
-1	0.139	-0.139
0	0.167	0
1	0.139	0.139
2	0.111	0.222
3	0.083	
4	0.056	
5	0.028	

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	-0.250
-2	0.111	-0.222
-1	0.139	-0.139
0	0.167	0
1	0.139	0.139
2	0.111	0.222
3	0.083	0.250
4	0.056	
5	0.028	

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	-0.250
-2	0.111	-0.222
-1	0.139	-0.139
0	0.167	0
1	0.139	0.139
2	0.111	0.222
3	0.083	0.250
4	0.056	0.222
5	0.028	

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	-0.250
-2	0.111	-0.222
-1	0.139	-0.139
0	0.167	0
1	0.139	0.139
2	0.111	0.222
3	0.083	0.250
4	0.056	0.222
5	0.028	0.139

Expected value: Examples

In Example 2:

$\mathbb{E}[X] =$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	-0.250
-2	0.111	-0.222
-1	0.139	-0.139
0	0.167	0
1	0.139	0.139
2	0.111	0.222
3	0.083	0.250
4	0.056	0.222
5	0.028	0.139
		$\mathbb{E}[X] = \sum P[x_i] \times x_i = 0$

Expected value: Examples

In Example 2:

$$\mathbb{E}[X] = -5 \times 0.03 - 4 \times 0.06 - 3 \times 0.08 - 2 \times 0.11 - 1 \times 0.14 + 0 \times 0.17 + 1 \times 0.14 + 2 \times 0.11 + 3 \times 0.08 + 4 \times 0.06 + 5 \times 0.03 = 0$$

x	$P(x)$	$P[x_i] \times x_i$
-5	0.028	-0.139
-4	0.056	-0.222
-3	0.083	-0.250
-2	0.111	-0.222
-1	0.139	-0.139
0	0.167	0
1	0.139	0.139
2	0.111	0.222
3	0.083	0.250
4	0.056	0.222
5	0.028	0.139
		$\mathbb{E}[X] = \sum P[x_i] \times x_i = 0$

Expected value: Examples

In Example 3:

$\mathbb{E}[X] =$

x	$P[x]$	$P[x_i] \times x_i$
0	0.05	
1	0.10	
2	0.20	
3	0.25	
4	0.15	
5	0.15	
6	0.10	

Expected value: Examples

In Example 3:

$\mathbb{E}[X] =$

x	$P[x]$	$P[x_i] \times x_i$
0	0.05	0.00
1	0.10	
2	0.20	
3	0.25	
4	0.15	
5	0.15	
6	0.10	

Expected value: Examples

In Example 3:

$\mathbb{E}[X] =$

x	$P[x]$	$P[x_i] \times x_i$
0	0.05	0.00
1	0.10	0.10
2	0.20	
3	0.25	
4	0.15	
5	0.15	
6	0.10	

Expected value: Examples

In Example 3:

$\mathbb{E}[X] =$

x	$P[x]$	$P[x_i] \times x_i$
0	0.05	0.00
1	0.10	0.10
2	0.20	0.40
3	0.25	
4	0.15	
5	0.15	
6	0.10	

Expected value: Examples

In Example 3:

$\mathbb{E}[X] =$

x	$P[x]$	$P[x_i] \times x_i$
0	0.05	0.00
1	0.10	0.10
2	0.20	0.40
3	0.25	0.75
4	0.15	
5	0.15	
6	0.10	

Expected value: Examples

In Example 3:

$\mathbb{E}[X] =$

x	$P[x]$	$P[x_i] \times x_i$
0	0.05	0.00
1	0.10	0.10
2	0.20	0.40
3	0.25	0.75
4	0.15	0.60
5	0.15	
6	0.10	

Expected value: Examples

In Example 3:

$\mathbb{E}[X] =$

x	$P[x]$	$P[x_i] \times x_i$
0	0.05	0.00
1	0.10	0.10
2	0.20	0.40
3	0.25	0.75
4	0.15	0.60
5	0.15	0.75
6	0.10	

Expected value: Examples

In Example 3:

$\mathbb{E}[X] =$

x	$P[x]$	$P[x_i] \times x_i$
0	0.05	0.00
1	0.10	0.10
2	0.20	0.40
3	0.25	0.75
4	0.15	0.60
5	0.15	0.75
6	0.10	0.60

Expected value: Examples

In Example 3:

$\mathbb{E}[X] =$

x	$P[x]$	$P[x_i] \times x_i$
0	0.05	0.00
1	0.10	0.10
2	0.20	0.40
3	0.25	0.75
4	0.15	0.60
5	0.15	0.75
6	0.10	0.60
		$\mathbb{E}[X] = \sum P[x_i] \times x_i = 3.20$

Expected value: Examples

In Example 3:

$$\mathbb{E}[X] = 0 \times 0.05 + 1 \times 0.10 + 2 \times 0.20 + 3 \times 0.25 + 4 \times 0.15 + 5 \times 0.15 + 6 \times 0.10 = 3.20$$

x	$P[x]$	$P[x_i] \times x_i$
0	0.05	0.00
1	0.10	0.10
2	0.20	0.40
3	0.25	0.75
4	0.15	0.60
5	0.15	0.75
6	0.10	0.60
		$\mathbb{E}[X] = \sum P[x_i] \times x_i = 3.20$

Expected value: Examples

In Example 4:

x	$P[x]$	$P[x_i] \times x_i$
0	0.67	
1	0.23	
5	0.08	
10	0.02	

Expected value: Examples

In Example 4:

x	$P[x]$	$P[x_i] \times x_i$
0	0.67	0.00
1	0.23	
5	0.08	
10	0.02	

Expected value: Examples

In Example 4:

x	$P[x]$	$P[x_i] \times x_i$
0	0.67	0.00
1	0.23	0.23
5	0.08	
10	0.02	

Expected value: Examples

In Example 4:

x	$P[x]$	$P[x_i] \times x_i$
0	0.67	0.00
1	0.23	0.23
5	0.08	0.38
10	0.02	

Expected value: Examples

In Example 4:

x	$P[x]$	$P[x_i] \times x_i$
0	0.67	0.00
1	0.23	0.23
5	0.08	0.38
10	0.02	0.19

Expected value: Examples

In Example 4:

$$\mathbb{E}[X] = 0 \times 0.67 + 1 \times 0.23 + 5 \times 0.08 + 10 \times 0.02 = 0.81$$

x	$P[x]$	$P[x_i] \times x_i$
0	0.67	0.00
1	0.23	0.23
5	0.08	0.38
10	0.02	0.19
		$\mathbb{E}[X] = \sum P[x_i] \times x_i = 0.81$

A researcher expects to get a grant of \$1000; \$10,000; \$100,000; and \$1,000,000 with probabilities 0.05; 0.4; 0.54; and 0.01. Calculate the expected value of her grant.

Expected value: Examples

A researcher expects to get a grant of \$1000; \$10,000; \$100,000; and \$1,000,000 with probabilities 0.05; 0.4; 0.54; and 0.01. Calculate the expected value of her grant.

x	$P[x]$	$x_i * P[x_i$
1000	0.05	50
10000	0.4	
100000	0.54	
1000000	0.01	
	1	

Expected value: Examples

A researcher expects to get a grant of \$1000; \$10,000; \$100,000; and \$1,000,000 with probabilities 0.05; 0.4; 0.54; and 0.01. Calculate the expected value of her grant.

x	$P[x]$	$x_i * P[x_i$
1000	0.05	50
10000	0.4	4000
100000	0.54	
1000000	0.01	
	1	

Expected value: Examples

A researcher expects to get a grant of \$1000; \$10,000; \$100,000; and \$1,000,000 with probabilities 0.05; 0.4; 0.54; and 0.01. Calculate the expected value of her grant.

x	$P[x]$	$x_i * P[x_i$
1000	0.05	50
10000	0.4	4000
100000	0.54	54000
1000000	0.01	
	1	

Expected value: Examples

A researcher expects to get a grant of \$1000; \$10,000; \$100,000; and \$1,000,000 with probabilities 0.05; 0.4; 0.54; and 0.01. Calculate the expected value of her grant.

x	$P[x]$	$x_i * P[x_i$
1000	0.05	50
10000	0.4	4000
100000	0.54	54000
1000000	0.01	10000
	1	

Expected value: Examples

A researcher expects to get a grant of \$1000; \$10,000; \$100,000; and \$1,000,000 with probabilities 0.05; 0.4; 0.54; and 0.01. Calculate the expected value of her grant.

x	$P[x]$	$x_i * P[x_i$
1000	0.05	50
10000	0.4	4000
100000	0.54	54000
1000000	0.01	10000
	1	$\mathbb{E}[X] = 68050$

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

Expected value: Examples

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x	$P[x]$	$x_i * P[x_i$

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000		

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000		
7000		

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000		
7000		
2000		

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000		
7000		
2000		
0		

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000	0.01	
7000		
2000		
0		

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000	0.01	
7000	0.05	
2000		
0		

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000	0.01	
7000	0.05	
2000	0.1	
0		

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000	0.01	
7000	0.05	
2000	0.1	
0	0.84	
	1	

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000	0.01	200
7000	0.05	
2000	0.1	
0	0.84	
	1	

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000	0.01	200
7000	0.05	350
2000	0.1	
0	0.84	
	1	

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000	0.01	200
7000	0.05	350
2000	0.1	200
0	0.84	
	1	

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000	0.01	200
7000	0.05	350
2000	0.1	200
0	0.84	0
	1	

Expected value: Examples

An insurance company offers to pay \$20000 when a car is totaled, \$7000 to cover severe damages, and \$2000 to cover minor damages. No payment is made when there is no damage. The company believes the likelihood of these scenarios to be: 0.01, 0.05, 0.1, and 0.84, respectively. What is the expected payout this company should anticipate to make to a policy holder?

x	$P[x]$	$x_i * P[x_i$
20000	0.01	200
7000	0.05	350
2000	0.1	200
0	0.84	0
	1	$\mathbb{E}[X] = 750$

Properties of the expected value

Let X and Y be random variables, and c be a constant.

- Constant: $\mathbb{E}[c] = c$ *if c is a constant*
- Constant Multiplication: $\mathbb{E}[cX] = c \times \mathbb{E}[X]$
- Constant Addition: $\mathbb{E}[X \pm c] = \mathbb{E}[X] \pm c$
- Addition: $\mathbb{E}[X \pm Y] = \mathbb{E}[X] \pm \mathbb{E}[Y]$
- Multiplication: $\mathbb{E}[X \times Y] = \mathbb{E}[X] \times \mathbb{E}[Y]$ *if X and Y are independent*

Properties of $\mathbb{E}$: Example

The average price of a cup of coffee, X, is \$1.5, while the average price of a muffin, y, is \$2.5. Their standard deviations are \$0.25 and \$0.5, respectively. If you get 2 cups of coffee and a muffin every day, what is your average daily spending?

$$\begin{aligned} \mathbb{E}[X] &= 1.5 \\ \mathbb{E}[Y] &= 2.5 \\ \mathbb{E}[2X + Y] &= 2\mathbb{E}[X] + \mathbb{E}[Y] \\ &= 2 \times 1.5 + 2.5 \\ &= 5.5 \end{aligned}$$

The Variance of a Random Variable

The variance, σ^2, of a random variable, X, is the expected value of the squared deviation between each value of the random variable and its mean, $\mathbb{E}[(x_i - \mathbb{E}[X])^2]$. In other words, it is the weighted average of squared deviations from the mean, weighted by probability of occurrence.

$$\sigma^2 = \mathbb{E}[(x_i - \mathbb{E}[X])^2] = \frac{\sum\limits_{i=1}^{n} P[x_i] \times (x_i - \mathbb{E}[X])^2}{\sum\limits_{i=1}^{n} P[x_i]} = \sum_{i=1}^{n} P[x_i] \times (x_i - \mathbb{E}[X])^2$$

The variance can also be written as:

$$\sigma^2 = \sum_{i=1}^{n} P[x_i] \times x_i^2 - [\mathbb{E}[X]]^2$$

Example: Variance Wait-times at the ER

The table below shows the distribution of wait times at the ER. Find the the variance and STDV.

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$$\sigma^2 = \sum_{i=1}^{10} P[x_i] \times x_i^2 - [\mathbb{E}[X]]^2 = 10508.5 - 100^2 = 508.5 \Rightarrow \sigma = 22.55$$

Variance, example

The number of vacancies at a Gulf Shore resort motel on a day during weekends through the tourist season has the following probability distribution:

Vacancies(X)	$P[x]$
0	0.1
1	0.2
2	0.3
3	0.15
4	0.12
5	0.08
6	0.05

Find the standard deviation of this random variable using the two methods we covered in class.

Variance, example

The standard deviation of this random variable using the two methods we covered in class is

x	$P[x]$	$P[x_i]x_i$	$P[x_i](x_i - \mathbb{E}[X])^2$	$P[x_i]x_i^2$
0	0.1			
1	0.2			
2	0.3			
3	0.15			
4	0.12			
5	0.08			
6	0.05			
$\sum$				

The standard deviation of this random variable using the two methods we covered in class is

x	$P[x]$	$P[x_i]x_i$	$P[x_i](x_i - \mathbb{E}[X])^2$	$P[x_i]x_i^2$
0	0.1	0		
1	0.2	0.2		
2	0.3	0.6		
3	0.15	0.45		
4	0.12	0.48		
5	0.08	0.4		
6	0.05	0.3		
$\sum$		2.43		
		$\mathbb{E}[X] = 2.43$		

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0	0.1	0	0.59049	
1	0.2	0.2	0.40898	
2	0.3	0.6	0.05547	
3	0.15	0.45	0.048735	
4	0.12	0.48	0.295788	
5	0.08	0.4	0.528392	
6	0.05	0.3	0.637245	
$\sum$		2.43	2.5651	
		$\mathbb{E}[X] = 2.43$	$\sigma^2 = 2.5651$	

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The standard deviation of this random variable using the two methods we covered in class is

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1	0.2	0.2	0.40898	0.2
2	0.3	0.6	0.05547	1.2
3	0.15	0.45	0.048735	1.35
4	0.12	0.48	0.295788	1.92
5	0.08	0.4	0.528392	2
6	0.05	0.3	0.637245	1.8
$\sum$		2.43	2.5651	8.47
		$\mathbb{E}[X] = 2.43$	$\sigma^2 = 2.5651$	$\sigma^2 = 8.47 - 2.43^2 = 2.5651$

Variance and Risk: Example

Consider two lotteries, A and B. With lottery A, there is a 20% chance that you will receive \$80, a 50% chance that you will receive \$40, and a 30% chance that you will receive \$10. With lottery B, there is a 40% chance that you will receive \$30, a 30% chance that you will receive \$40, and a 30% chance that you will receive \$50. Compare the return on these two lotteries.

Variance and Risk: Example

To compare these lotteries we need to know their average payoff and its standard deviation.

A	B	$P[A]$	$P[B]$	$A * P[A]$	$B * P[B]$	$P[A](A - \mathbb{E}[A])^2$	$P[B](B - \mathbb{E}[B])^2$
80	30	0.2	0.4				
40	40	0.5	0.3				
10	50	0.3	0.3				

Variance and Risk: Example

To compare these lotteries we need to know their average payoff and its standard deviation.

A	B	$P[A]$	$P[B]$	$A * P[A]$	$B * P[B]$	$P[A](A - \mathbb{E}[A])^2$	$P[B](B - \mathbb{E}[B])^2$
80	30	0.2	0.4	16			
40	40	0.5	0.3	20			
10	50	0.3	0.3	3			
				$\mathbb{E}[A] = 39$			

Variance and Risk: Example

To compare these lotteries we need to know their average payoff and its standard deviation.

A	B	$P[A]$	$P[B]$	$A * P[A]$	$B * P[B]$	$P[A](A - \mathbb{E}[A])^2$	$P[B](B - \mathbb{E}[B])^2$
80	30	0.2	0.4	16	12		
40	40	0.5	0.3	20	12		
10	50	0.3	0.3	3	15		
				$\mathbb{E}[A] = 39$	$\mathbb{E}[B] = 39$		

Variance and Risk: Example

To compare these lotteries we need to know their average payoff and its standard deviation.

A	B	$P[A]$	$P[B]$	$A * P[A]$	$B * P[B]$	$P[A](A - \mathbb{E}[A])^2$	$P[B](B - \mathbb{E}[B])^2$
80	30	0.2	0.4	16	12	336.2	
40	40	0.5	0.3	20	12	0.5	
10	50	0.3	0.3	3	15	252.3	
				$\mathbb{E}[A] = 39$	$\mathbb{E}[B] = 39$	$\sigma_A^2 = 589$	
						$\sigma_A = 24.27$	

Variance and Risk: Example

To compare these lotteries we need to know their average payoff and its standard deviation.

A	B	$P[A]$	$P[B]$	$A * P[A]$	$B * P[B]$	$P[A](A - \mathbb{E}[A])^2$	$P[B](B - \mathbb{E}[B])^2$
80	30	0.2	0.4	16	12	336.2	32.4
40	40	0.5	0.3	20	12	0.5	0.3
10	50	0.3	0.3	3	15	252.3	36.3
				$\mathbb{E}[A] = 39$	$\mathbb{E}[B] = 39$	$\sigma_A^2 = 589$	$\sigma_B^2 = 69$
						$\sigma_A = 24.27$	$\sigma_B = 8.31$

Properties of Variance

Let X and Y be random variables, and c be a constant.

- Constant: $Var(c) = 0$
- Constant Multiplication;$Var(cX) = c^2 Var(X)$
- Constant Addition: $Var(X + c) = Var(X)$
- Addition: $Var(X + Y) = Var(X) + Var(Y)$ if X and y are independent.
- Subtraction: $Var(X - Y) = Var(X) + Var(Y)$ if X and Y are independent.

Properties of the variance: Example

The average price of a cup of coffee, X, is \$1.5, while the average price of a muffin, y, is \$2.5. Their standard deviations are \$0.25 and \$0.5, respectively. If you get 2 cups of coffee and a muffin every day, what is your average daily spending? And standard deviation? Assume that the price of coffee and muffins are independent.

$$\begin{aligned} \mathbb{E}[X] &= 1.5 \\ \mathbb{E}[Y] &= 2.5 \\ \mathbb{E}[2X+Y] &= \\ &= \\ &= \end{aligned} \qquad \begin{aligned} Var(X) &= 0.25^2 = 0.0625 \\ Var(Y) &= 0.5^2 = 0.25 \\ Var(2X+Y) &= \\ &= \end{aligned}$$

Properties of the variance: Example

The average price of a cup of coffee, X, is \$1.5, while the average price of a muffin, y, is \$2.5. Their standard deviations are \$0.25 and \$0.5, respectively. If you get 2 cups of coffee and a muffin every day, what is your average daily spending? And standard deviation? Assume that the price of coffee and muffins are independent.

$$\begin{aligned} \mathbb{E}[X] &= 1.5 \\ \mathbb{E}[Y] &= 2.5 \\ \mathbb{E}[2X+Y] &= 2\mathbb{E}[X] + \mathbb{E}[Y] \\ &= 2 \times 1.5 + 2.5 \\ &= 5.5 \end{aligned}$$

$$\begin{aligned} Var(X) &= 0.25^2 = 0.0625 \\ Var(Y) &= 0.5^2 = 0.25 \\ Var(2X+Y) &= \\ &= \end{aligned}$$

Properties of the variance: Example

The average price of a cup of coffee, X, is \$1.5, while the average price of a muffin, y, is \$2.5. Their standard deviations are \$0.25 and \$0.5, respectively. If you get 2 cups of coffee and a muffin every day, what is your average daily spending? And standard deviation? Assume that the price of coffee and muffins are independent.

$$
\begin{aligned}
\mathbb{E}[X] &= 1.5 & Var(X) &= 0.25^2 = 0.0625 \\
\mathbb{E}[Y] &= 2.5 & Var(Y) &= 0.5^2 = 0.25 \\
\mathbb{E}[2X+Y] &= 2\mathbb{E}[X] + \mathbb{E}[Y] & Var(2X+Y) &= 4Var(X) + Var(Y) \\
&= 2 \times 1.5 + 2.5 & &= 4 \times 0.0626 + 0.25 = 0.5 \\
&= 5.5 & &\Rightarrow SDV(2X+Y) = \sqrt{0.5} \\
& & &\Rightarrow SDV(2X+Y) = 0.707
\end{aligned}
$$

Special Discrete Prob. Distributions

There are some convenient distributions for which we already know the mean, the standard deviation, and a method of easily calculating probabilities.

- The Uniform Distribution
- Bernoulli Distribution
- Binomial Distribution
- Geometric Distribution
- Hypergeometric Distribution
- Poisson Distribution

The Uniform distribution

Definition

We say a discrete random variable X has a ***uniform distribution*** over the interval $[a, b]$, if all the values X takes on are equally likely. That is
$P[x_1] = P[x_2] = P[x_3] = P[x_i], \forall x_i \in [a, b]$.
If we let n be the number of values X takes on, then:

- $P[x_i] = \frac{1}{n}, \forall x_i \in x$,
- $\mathbb{E}[X] = \frac{a+b}{2}$,
- $\sigma^2 = \frac{(b-a+1)^2-1}{12}$.

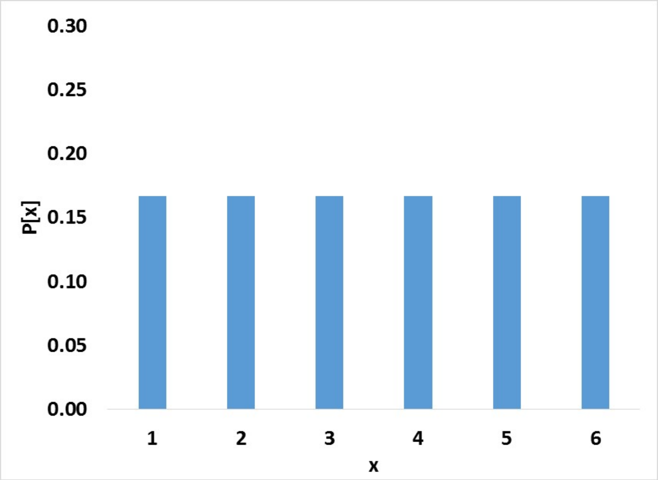
Uniform Distribution: six-sided die example

Let X represent the outcome of rolling a six-sided die.
$P[X = 1] = P[X = 2] = \ldots = P[X = 6] = \frac{1}{6}$

x	$P[x]$	$x_i P[x_i]$	$P[x_i](x_i - \mathbb{E}[X])^2$
1	0.167	0.167	1.042
2	0.167	0.333	0.375
3	0.167	0.500	0.042
4	0.167	0.667	0.042
5	0.167	0.833	0.375
6	0.167	1	1.042
$\sum$		3.5	2.917

$$\begin{aligned}
\mathbb{E}[X] &= \frac{a+b}{2} \\
&= \frac{1+6}{2} = 3.5 \\
\sigma^2 &= \frac{(b-a+1)^2 - 1}{12} \\
&= \frac{(6-1+1)^2 - 1}{12} \\
&= \frac{35}{12} = 2.917
\end{aligned}$$

Uniform Distribution: six-sided die example



Uniform Distribution: Example 2

Let $X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and suppose each possible value has equal probability. Find, $P[X = 4]$, $P[X = 7]$, $\mathbb{E}[X]$ and σ_X

Answer:

Uniform Distribution: Example 2

Let $X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and suppose each possible value has equal probability. Find, $P[X = 4]$, $P[X = 7]$, $\mathbb{E}[X]$ and σ_X

Answer: $P[X = 4] = P[X = 7] = P[X = x_i] = \frac{1}{10}$.

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Let $X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and suppose each possible value has equal probability. Find, $P[X = 4]$, $P[X = 7]$, $\mathbb{E}[X]$ and σ_X

Answer: $P[X = 4] = P[X = 7] = P[X = x_i] = \frac{1}{10}$.

To find $\mathbb{E}[X]$, notice that $a = 0$ and $b = 9$. So,

$$\mathbb{E}[X] = \frac{0+9}{2} = 4.5$$

And,

Uniform Distribution: Example 2

Let $X = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and suppose each possible value has equal probability. Find, $P[X = 4]$, $P[X = 7]$, $\mathbb{E}[X]$ and σ_X

Answer: $P[X = 4] = P[X = 7] = P[X = x_i] = \frac{1}{10}$.

To find $\mathbb{E}[X]$, notice that $a = 0$ and $b = 9$. So,

$$\mathbb{E}[X] = \frac{0 + 9}{2} = 4.5$$

And,

$$\sigma_X = \sqrt{\frac{(9 - 0 + 1)^2 - 1}{12}} = \sqrt{8.25} = 2.872$$

Uniform Distribution: Example 3

Let X represent the outcome of rolling a fair 12-sided die that is numbered -6 through 5. Find, $P[X = 0]$, $P[X = -3]$, $\mathbb{E}[X]$ and σ_X

Answer:

Uniform Distribution: Example 3

Let X represent the outcome of rolling a fair 12-sided die that is numbered -6 through 5. Find, $P[X = 0]$, $P[X = -3]$, $\mathbb{E}[X]$ and σ_X

Answer: $P[X = 0] = P[X = -3] = P[X = x_i] = \frac{1}{12}$.

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Let X represent the outcome of rolling a fair 12-sided die that is numbered -6 through 5. Find, $P[X = 0]$, $P[X = -3]$, $\mathbb{E}[X]$ and σ_X

Answer: $P[X = 0] = P[X = -3] = P[X = x_i] = \frac{1}{12}$.

To find $\mathbb{E}[X]$, notice that $a = -6$ and $b = 5$. So,

$$\mathbb{E}[X] = \frac{-6 + 5}{2} = -0.5$$

And,

Uniform Distribution: Example 3

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Answer: $P[X = 0] = P[X = -3] = P[X = x_i] = \frac{1}{12}$.

To find $\mathbb{E}[X]$, notice that $a = -6$ and $b = 5$. So,

$$\mathbb{E}[X] = \frac{-6 + 5}{2} = -0.5$$

And,

$$\sigma_X = \sqrt{\frac{(5 - (-6) + 1)^2 - 1}{12}} = \sqrt{11.916667} = 3.4521$$

Bernoulli Distribution

Definition

The ***Bernoulli distribution*** is a discrete distribution having two possible outcomes, $X = 1$ and $X = 0$. The outcomes are usually "labeled" success and failure, with p denoting the possibility of success and $q = 1 - p$ denoting the probability of failure.

$$X = \begin{cases} 1 & \text{with probability } P[X = 1] = p \\ 0 & \text{with probability } P[X = 0] = q = 1 - p \end{cases}$$

$\mathbb{E}[X] = \sum_{i=1}^{2} P[x_i] \times x_i = p \times 1 + q \times 0 = p$

$$\sigma^2 = \sum_{i=1}^{2} P[x_i] \times x_i^2 - [\mathbb{E}[X]]^2 = q \times 0^2 + p \times 1^2 - p^2 = p - p^2$$

$$\sigma^2 = p(1 - p) = pq \Rightarrow \sigma = \sqrt{pq}$$

Bernoulli Distribution: Example

Historically, it rains 121 days a year in NYC. Let X be a random variable representing whether or not it will rain tomorrow.

$$X = \begin{cases} 1 & \text{with probability } p = \frac{121}{365} \\ 0 & \text{with probability } q = 1 - p = \frac{244}{365} \end{cases}$$

What is the expected value of X? What is the standard deviation of X?

Answer:

Bernoulli Distribution: Example

Historically, it rains 121 days a year in NYC. Let X be a random variable representing whether or not it will rain tomorrow.

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What is the expected value of X? What is the standard deviation of X?

Answer:

$$\mathbb{E}[X] = p = \frac{121}{365} = 0.3315$$

and,

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What is the expected value of X? What is the standard deviation of X?

Answer:

$$\mathbb{E}[X] = p = \frac{121}{365} = 0.3315$$

and,

$$\sigma_X = \sqrt{p * q} = \sqrt{\frac{121}{365} \times \frac{244}{365}} = 0.2216$$

Successive Bernoulli Trials: Example

Suppose a cooler has 20 cans of coke, C, and 10 of Pepsi, P. Suppose we draw one can, with replacement, five times. Let X be a random variable representing the number of coke cans. What is the probability distribution of X?

Successive Bernoulli Trials: Example

X be a random variable representing the number of coke cans, what values can it take?

.

Successive Bernoulli Trials: Example

X be a random variable representing the number of coke cans, what values can it take?

.

x	$P[x]$
0	
1	
2	
3	
4	
5	

Successive Bernoulli Trials: Example

X be a random variable representing the number of coke cans, what values can it take?

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x	$P[x]$
0	
1	
2	
3	
4	
5	

CPPPP PCPPP PPCPP PPPCP PPPPC

Successive Bernoulli Trials: Example

X be a random variable representing the number of coke cans, what values can it take?

.

x	$P[x]$
0	
1	$\left(\frac{2}{3}\right)^1$
2	
3	
4	
5	

CPPPP PCPPP PPCPP PPPCP PPPPC

Successive Bernoulli Trials: Example

X be a random variable representing the number of coke cans, what values can it take?

.

x	$P[x]$				
0					
1	C_1^5	$\left(\frac{2}{3}\right)^1$	$\left(\frac{1}{3}\right)^4$	$= 0.0412$	CPPPP PCPPP PPCPP PPPCP PPPPC
2					
3					
4					
5					

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1	C_1^5	$\left(\frac{2}{3}\right)^1$	$\left(\frac{1}{3}\right)^4$	$= 0.0412$	CPPPP PCPPP PPCPP PPPCP PPPPC
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2					
3					
4					
5					

Successive Bernoulli Trials: Example

X be a random variable representing the number of coke cans, what values can it take?

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x	$P[x]$				
0	C_0^5	$\left(\frac{2}{3}\right)^0$	$\left(\frac{1}{3}\right)^5$	$= 0.0041$	PPPPP
1	C_1^5	$\left(\frac{2}{3}\right)^1$	$\left(\frac{1}{3}\right)^4$	$= 0.0412$	CPPPP PCPPP PPCPP PPPCP PPPPC
2					PPPCC PPCPC PPCCP PCPPC PCPCP
3					PCCPP CPPPC CPPCP CPCPP CCPPP
4					
5					

Successive Bernoulli Trials: Example

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1	C_1^5	$\left(\frac{2}{3}\right)^1$	$\left(\frac{1}{3}\right)^4$	$= 0.0412$	CPPPP PCPPP PPCPP PPPCP PPPPC
2	C_2^5	$\left(\frac{2}{3}\right)^2$	$\left(\frac{1}{3}\right)^3$	$=0.1646$	PPPCC PPCPC PPCCP PCPPC PCPCP
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4					
5					

Successive Bernoulli Trials: Example

X be a random variable representing the number of coke cans, what values can it take?

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3				
4				
5				

PPPPP

CPPPP PCPPP PPCPP PPPCP PPPPC

PPPCC PPCPC PPCCP PCPPC PCPCP

PCCPP CPPPC CPPCP CPCPP CCPPP

PPCCC PCPCC PCCPC PCCCP CPPCC

CPCPC CPCCP CCPPC CCPCP CCCPP

Successive Bernoulli Trials: Example

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2	C_2^5	$\left(\frac{2}{3}\right)^2$	$\left(\frac{1}{3}\right)^3$	$=0.1646$	PPPCC PPCPC PPCCP PCPPC PCPCP
3	C_3^5	$\left(\frac{2}{3}\right)^3$	$\left(\frac{1}{3}\right)^2$	$=0.3292$	PCCPP CPPPC CPPCP CPCPP CCPPP
4					PPCCC PCPCC PCCPC PCCCP CPPCC
5					CPCPC CPCCP CCPPC CCPCP CCCPP

Successive Bernoulli Trials: Example

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4	C_4^5	$\left(\frac{2}{3}\right)^4$	$\left(\frac{1}{3}\right)^1$	$=0.3292$	PCCCC CPCCC CCPCC CCCPC CCCCP
5					

Successive Bernoulli Trials: Example

X be a random variable representing the number of coke cans, what values can it take?

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x		$P[x]$			
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1	C_1^5	$\left(\frac{2}{3}\right)^1$	$\left(\frac{1}{3}\right)^4$	$= 0.0412$	CPPPP PCPPP PPCPP PPPCP PPPPC
2	C_2^5	$\left(\frac{2}{3}\right)^2$	$\left(\frac{1}{3}\right)^3$	$=0.1646$	PPPCC PPCPC PPCCP PCPPC PCPCP PCCPP CPPPC CPPCP CPCPP CCPPP
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4	C_4^5	$\left(\frac{2}{3}\right)^4$	$\left(\frac{1}{3}\right)^1$	$=0.3292$	PCCCC CPCCC CCPCC CCCPC CCCCP
5					CCCCC

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3	C_3^5	$\left(\frac{2}{3}\right)^3$	$\left(\frac{1}{3}\right)^2$	$=0.3292$	PPCCC PCPCC PCCPC PCCCP CPPCC CPCPC CPCCP CCPPC CCPCP CCCPP
4	C_4^5	$\left(\frac{2}{3}\right)^4$	$\left(\frac{1}{3}\right)^1$	$=0.3292$	PCCCC CPCCC CCPCC CCCPC CCCCP
5	C_5^5	$\left(\frac{2}{3}\right)^5$	$\left(\frac{1}{3}\right)^0$	$= 0.1317$	CCCCC

The Binomial Distribution

The binomial distribution is based on two or more successive Bernoulli trials satisfying:

- In each trial, there are just two possible outcomes, usually denoted as success or failure.
- The trials are statistically independent; that is, the outcome in any trial is not affected by the outcomes of earlier trials, and it does not affect the outcomes of later trials.
- The probability of a success remains the same from one trial to the next.

The Binomial Distribution

Definition

When a random variable X represents the number of successes in n Bernoulli trails with probability of success p and probability of failure q, we say that X has a ***binomial distribution***, and denote $X \sim \text{Bin}(n, p)$. The probability of any number of successes x can be expressed as:

$$P[X = x] = \frac{n!}{x!(n-x)!} \times p^x \times q^{n-x} = C_x^n \times p^x \times q^{n-x}$$

The expected value of X is $\mathbb{E}[X] = n \times p$
The Variance of X is $\sigma^2 = n \times p \times q$

The binomial distribution: Example 1

20% of part-time workers participate in retirement benefits. Ten part-time workers are randomly selected. Let X be the number of persons in this group who participate in retirement benefits. What's the probability distribution of x?

The binomial distribution: Example 1

20% of part-time workers participate in retirement benefits. Ten part-time workers are randomly selected. Let X be the number of persons in this group who participate in retirement benefits. What's the probability distribution of x?

Answer

- We have 10 Bernoulli trails. In each trial, a person either participates in retirement benefits, with probability $p = 0.2$, or not, with probability $q = 1 - p = 0.8$.

$$\text{A worker either} \begin{cases} \text{Paticipates } p = 0.2 \\ \text{Doesn't participate } q = 1 - p = 0.8 \end{cases}$$

The binomial distribution: Example 1, Contd.

- The random variable X represents the number of successes in the $n = 10$ Bernoulli trails. X can take on any of the values $\{0, 1, 2, 3, ..., 9, 10\}$

The binomial distribution: Example 1, Contd.

- The random variable X represents the number of successes in the $n = 10$ Bernoulli trails. X can take on any of the values $\{0, 1, 2, 3, ..., 9, 10\}$
- We can calculate the probability that X takes any of those values using the binomial distribution function:

$$P[X = x] = C_x^n \times p^x \times q^{n-x}$$

The binomial distribution: Example 1, Contd.

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- We can calculate the probability that X takes any of those values using the binomial distribution function:

$$P[X = x] = C_x^n \times p^x \times q^{n-x}$$

- So, the probability that $X = 0$ is:

$$\begin{aligned} P[X = 0] &= C_0^{10} \times p^0 \times q^{10-0} \\ &= \frac{10!}{0! * (10-0)!} * (0.2)^0 (0.8)^{10} \\ &= 0.107 \end{aligned}$$

The binomial distribution: Example 1, Contd.

- The probability that $X = 1$ is:

The binomial distribution: Example 1, Contd.

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The binomial distribution: Example 1, Contd.

- The probability that $X = 1$ is:

$$\begin{aligned} P[X = 1] &= C_1^{10} \times p^1 \times q^{10-1} \\ &= \frac{10!}{1! * (10-1)!} * (0.2)^1 (0.8)^9 \\ &= 0.268 \end{aligned}$$

- The probability that $X = 2$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 1$ is:

$$\begin{aligned} P[X = 1] &= C_1^{10} \times p^1 \times q^{10-1} \\ &= \frac{10!}{1! * (10-1)!} * (0.2)^1 (0.8)^9 \\ &= 0.268 \end{aligned}$$

- The probability that $X = 2$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 1$ is:

$$P[X=1] = C_1^{10} \times p^1 \times q^{10-1}$$
$$= \frac{10!}{1! * (10-1)!} * (0.2)^1 (0.8)^9$$
$$= 0.268$$

- The probability that $X = 2$ is:

$$P[X=2] = C_2^{10} \times p^2 \times q^{10-2}$$
$$= \frac{10!}{2! * (10-2)!} * (0.2)^2 (0.8)^8$$
$$= 0.302$$

The binomial distribution: Example 1, Contd.

- The probability that $X = 3$ is:

The binomial distribution: Example 1, Contd.

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The binomial distribution: Example 1, Contd.

- The probability that $X = 3$ is:

$$\begin{aligned} P[X=3] &= C_3^{10} \times p^3 \times q^{10-3} \\ &= \frac{10!}{3! * (10-3)!} * (0.2)^3 (0.8)^7 \\ &= 0.201 \end{aligned}$$

- The probability that $X = 4$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 3$ is:

$$\begin{aligned} P[X=3] &= C_3^{10} \times p^3 \times q^{10-3} \\ &= \frac{10!}{3! * (10-3)!} * (0.2)^3 (0.8)^7 \\ &= 0.201 \end{aligned}$$

- The probability that $X = 4$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 3$ is:

$$\begin{aligned} P[X=3] &= C_3^{10} \times p^3 \times q^{10-3} \\ &= \frac{10!}{3! * (10-3)!} * (0.2)^3 (0.8)^7 \\ &= 0.201 \end{aligned}$$

- The probability that $X = 4$ is:

$$\begin{aligned} P[X=4] &= C_4^{10} \times p^4 \times q^{10-4} \\ &= \frac{10!}{4! * (10-4)!} * (0.2)^4 (0.8)^6 \\ &= 0.088 \end{aligned}$$

The binomial distribution: Example 1, Contd.

- The probability that $X = 5$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 5$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 5$ is:

$$P[X=5] = C_5^{10} \times p^5 \times q^{10-5}$$
$$= \frac{10!}{5! * (10-5)!} * (0.2)^5 (0.8)^5$$
$$= 0.026$$

- The probability that $X = 6$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 5$ is:

$$\begin{aligned} P[X=5] &= C_5^{10} \times p^5 \times q^{10-5} \\ &= \frac{10!}{5! * (10-5)!} * (0.2)^5 (0.8)^5 \\ &= 0.026 \end{aligned}$$

- The probability that $X = 6$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 5$ is:

$$\begin{aligned} P[X=5] &= C_5^{10} \times p^5 \times q^{10-5} \\ &= \frac{10!}{5! * (10-5)!} * (0.2)^5 (0.8)^5 \\ &= 0.026 \end{aligned}$$

- The probability that $X = 6$ is:

$$\begin{aligned} P[X=6] &= C_6^{10} \times p^6 \times q^{10-6} \\ &= \frac{10!}{6! * (10-6)!} * (0.2)^6 (0.8)^4 \\ &= 0.006 \end{aligned}$$

The binomial distribution: Example 1, Contd.

- The probability that $X = 7$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 7$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 7$ is:

$$\begin{aligned} P[X = 7] &= C_7^{10} \times p^7 \times q^{10-7} \\ &= \frac{10!}{7! * (10-7)!} * (0.2)^7 (0.8)^3 \\ &= 0.001 \end{aligned}$$

- The probability that $X = 8$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 7$ is:

$$\begin{aligned} P[X=7] &= C_7^{10} \times p^7 \times q^{10-7} \\ &= \frac{10!}{7! * (10-7)!} * (0.2)^7 (0.8)^3 \\ &= 0.001 \end{aligned}$$

- The probability that $X = 8$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 7$ is:

$$\begin{aligned} P[X = 7] &= C_7^{10} \times p^7 \times q^{10-7} \\ &= \frac{10!}{7! * (10-7)!} * (0.2)^7 (0.8)^3 \\ &= 0.001 \end{aligned}$$

- The probability that $X = 8$ is:

$$\begin{aligned} P[X = 8] &= C_8^{10} \times p^8 \times q^{10-8} \\ &= \frac{10!}{8! * (10-8)!} * (0.2)^8 (0.8)^2 \\ &= 0.00007 \end{aligned}$$

The binomial distribution: Example 1, Contd.

- The probability that $X = 9$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 9$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 9$ is:

$$\begin{aligned} P[X = 9] &= C_9^{10} \times p^9 \times q^{10-9} \\ &= \frac{10!}{9! * (10-9)!} * (0.2)^9 (0.8)^1 \\ &= 0.0000041 \end{aligned}$$

- The probability that $X = 10$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 9$ is:

$$\begin{aligned} P[X = 9] &= C_9^{10} \times p^9 \times q^{10-9} \\ &= \frac{10!}{9! * (10-9)!} * (0.2)^9 (0.8)^1 \\ &= 0.0000041 \end{aligned}$$

- The probability that $X = 10$ is:

The binomial distribution: Example 1, Contd.

- The probability that $X = 9$ is:

$$\begin{aligned} P[X = 9] &= C_9^{10} \times p^9 \times q^{10-9} \\ &= \frac{10!}{9! * (10-9)!} * (0.2)^9 (0.8)^1 \\ &= 0.0000041 \end{aligned}$$

- The probability that $X = 10$ is:

$$\begin{aligned} P[X = 10] &= C_{10}^{10} \times p^{10} \times q^{10-10} \\ &= \frac{10!}{10! * (10-10)!} * (0.2)^{10} (0.8)^0 \\ &= 0.0000001 \end{aligned}$$

So, the distribution of X is:

x	$P[x]$
0	0.1074
1	0.2684
2	0.3020
3	0.2013
4	0.0881
5	0.0264
6	0.0055
7	0.0008
8	0.0001
9	0.0000
10	0.0000
$\sum$	1

$$\begin{aligned}\mathbb{E}[X] &= n * p = 10 * 0.2 = 2\\ \sigma_X &= \sqrt{npq}\\ &= \sqrt{10 * 0.2 * 0.8} = 1.2649\end{aligned}$$

The binomial distribution: Example 2

There is about 0.5% chance that two chicks hatch from the same egg. What is the probability that 3 out of 100 randomly chosen eggs are carrying twin chicks? Whats the probability that more than 5 eggs are carrying twin chicks?

Answer:

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Answer:

$$P[X = 3] =$$

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Answer:

$$P[X = 3] = C_3^{100} \times p^3 \times q^{100-3} =$$

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Answer:

$$P[X = 3] = C_3^{100} \times p^3 \times q^{100-3} = 0.012429649$$

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$$P[X > 5] =$$

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$$P[X > 5] = 1 - P[X \leq 5]$$

$$= 1 - (P[X = 0] + P[X = 1] + P[X = 2] + P[X = 3] + P[X = 4] + P[X = 5])$$

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$$= 1 - (P[X = 0] + P[X = 1] + P[X = 2] + P[X = 3] + P[X = 4] + P[X = 5])$$

$$= 0.0000125$$

More Binomial Dist. Examples

1. Suppose $X \sim \text{Bin}(4, 0.5)$, What is $P[X < 2]$, $\mathbb{E}[X]$, σ_X?

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Answ $P[X < 2] = P[X = 0] + P[X = 1] = 0.0625 + 0.25 = 0.3125$
$\mathbb{E}[X] = 4 * 0.5 = 2$
$\sigma_X = \sqrt{4 * 0.5 * 0.5} = 1$

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2. Suppose $X \sim \text{Bin}(20, 0.1)$, What is $P[X = 10]$, $\mathbb{E}[X]$, σ_X?

Answ $P[X = 10] = 0.00006$
$\mathbb{E}[X] = 20 * 0.1 = 2$
$\sigma_X = \sqrt{20 * 0.1 * 0.9} = 1.3416$

More Binomial Dist. Examples

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3. Suppose $X \sim \text{Bin}(50, 0.25)$, What is $P[X \geq 2]$, $\mathbb{E}[X]$, σ_X?

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3. Suppose $X \sim \text{Bin}(50, 0.25)$, What is $P[X \geq 2]$, $\mathbb{E}[X]$, σ_X?

Answ $P[X \geq 2] = 1 - P[X < 2] = 0.9999$
$\mathbb{E}[X] = 50 * 0.25 = 12.5$
$\sigma_X = \sqrt{50 * 0.25 * 0.75} = 3.0619$

The shape of the binomial distribution

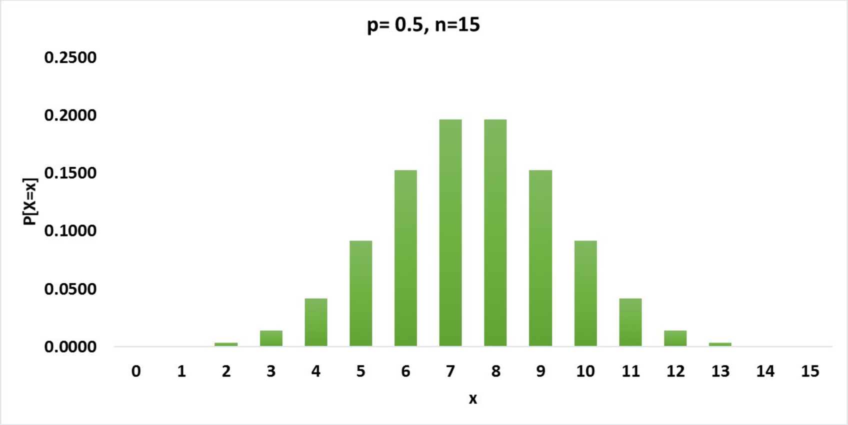
Suppose we draw a group of 15 Fordham students at random with replacement. Let X be the number of males in this group, and assume that half of Fordham students are males. Let Y be the number of students with a scholarship in this group, and assume 30% of Fordham students have a scholarship. Finally, let Z be the number of students in this group who have a student loan, and assume that 70% of Fordham students have student loans. Compare the probability distributions of X, Y, and Z.

The shape of the binomial distribution: X

x	$P[X = x]$
0	0.0000
1	0.0005
2	0.0032
3	0.0139
4	0.0417
5	0.0916
6	0.1527
7	0.1964
8	0.1964
9	0.1527
10	0.0916
11	0.0417
12	0.0139
13	0.0032
14	0.0005
15	0.0000

$$
\begin{aligned}
n &= 15 \\
p &= 0.5 \\
\mathbb{E}[X] &= n * p = 15 * 0.5 = 7.5 \\
\sigma_X &= \sqrt{npq} \\
&= \sqrt{15 * 0.5 * 0.5} = 1.93649
\end{aligned}
$$

The shape of the binomial distribution: X

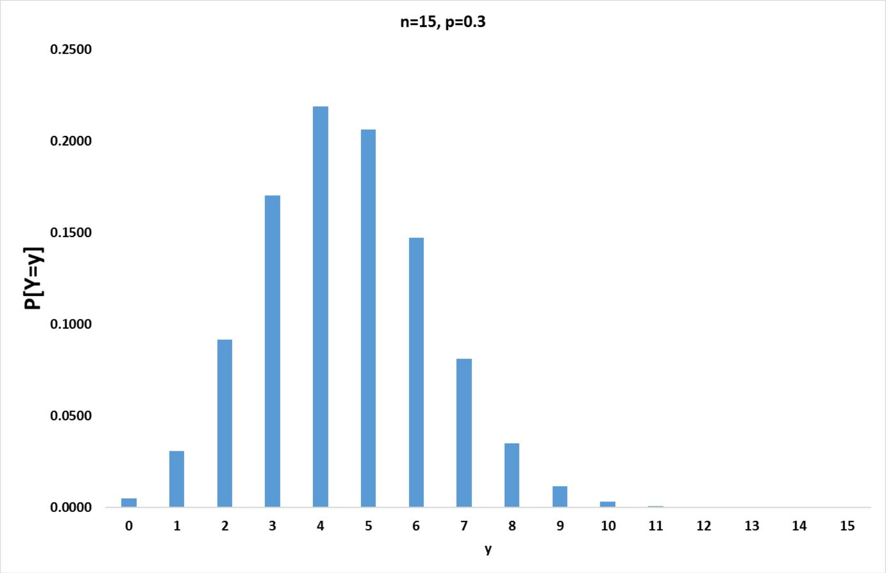


The shape of the binomial distribution: Y

y	$P[Y = y]$
0	0.0047
1	0.0305
2	0.0916
3	0.1700
4	0.2186
5	0.2061
6	0.1472
7	0.0811
8	0.0348
9	0.0116
10	0.0030
11	0.0006
12	0.0001
13	0.0000
14	0.0000
15	0.0000

$$\begin{aligned} n &= 15 \\ p &= 0.3 \\ \mathbb{E}[Y] &= n * p = 15 * 0.3 = 4.5 \\ \sigma_Y &= \sqrt{npq} \\ &= \sqrt{15 * 0.3 * 0.7} = 1.7748 \end{aligned}$$

The shape of the binomial distribution: Y

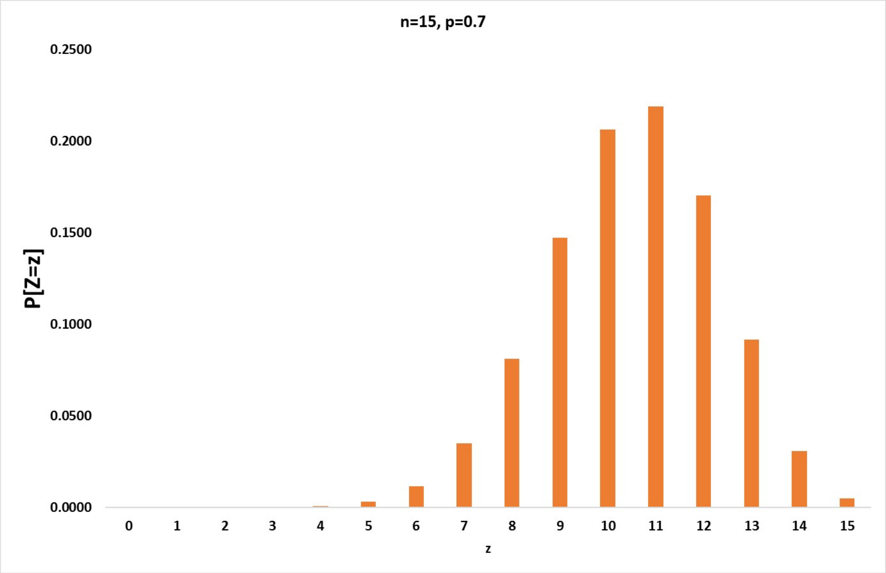


The shape of the binomial distribution: Z

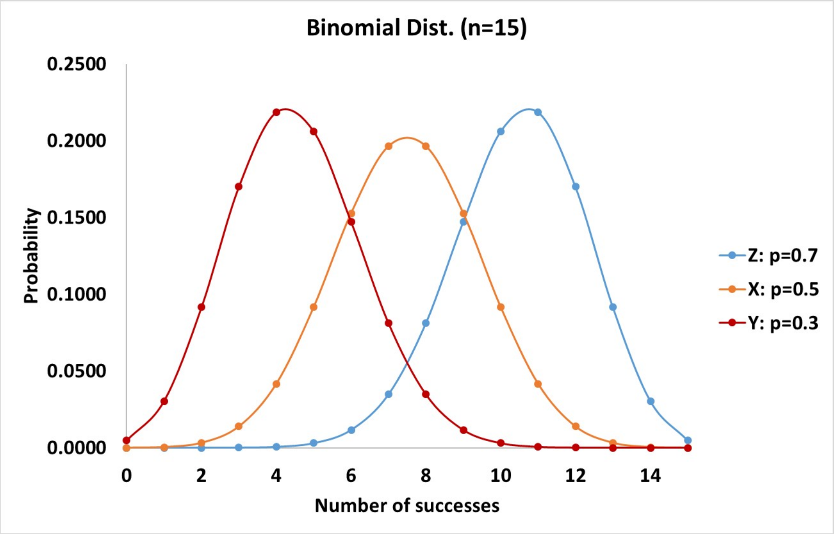
z	$P[Z = z]$
0	0.0047
1	0.0305
2	0.0916
3	0.1700
4	0.2186
5	0.2061
6	0.1472
7	0.0811
8	0.0348
9	0.0116
10	0.0030
11	0.0006
12	0.0001
13	0.0000
14	0.0000
15	0.0000

$$
\begin{aligned}
n &= 15 \\
p &= 0.7 \\
\mathbb{E}[Z] &= n * p = 15 * 0.7 = 10.5 \\
\sigma_Z &= \sqrt{npq} \\
&= \sqrt{15 * 0.7 * 0.3} = 1.7748
\end{aligned}
$$

The shape of the binomial distribution: Z



Comparing Binomial Distributions: X, Y, & Z



Binomial Tables

To avoid cumbersome calculations, for every n Bernoulli trials, we can construct a table of the probability that the number of success X equals k, $P[X = k]$. The number k is any value that the random variable X can take. The same can be done for $P[X \leq k]$.

Binomial Distribution, Individual Probabilities for x = number of successes in n trials, prob($x = k$)

$n = 2$

π:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
k: 0	0.8100	0.6400	0.4900	0.3600	0.2500	0.1600	0.0900	0.0400	0.0100
1	0.1800	0.3200	0.4200	0.4800	0.5000	0.4800	0.4200	0.3200	0.1800
2	0.0100	0.0400	0.0900	0.1600	0.2500	0.3600	0.4900	0.6400	0.8100

Binomial Distribution, Cumulative Probabilities for x = number of successes in n trials, prob($x \leq k$)

$n = 2$

π:	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
k: 0	0.8100	0.6400	0.4900	0.3600	0.2500	0.1600	0.0900	0.0400	0.0100
1	0.9900	0.9600	0.9100	0.8400	0.7500	0.6400	0.5100	0.3600	0.1900
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000

Using Binomial Tables: Example 1

A ball is drawn from an urn containing 3 white and 3 black balls. After the ball is drawn, it is then replaced and another ball is drawn. This goes on indefinitely. What is the probability that of the first 4 balls drawn, exactly 2 are white? Use the binomial distribution table to answer this question.

Answer::

Using Binomial Tables: Example 1

A ball is drawn from an urn containing 3 white and 3 black balls. After the ball is drawn, it is then replaced and another ball is drawn. This goes on indefinitely. What is the probability that of the first 4 balls drawn, exactly 2 are white? Use the binomial distribution table to answer this question.

Answer:: Here $n = 4$, $p = \frac{1}{2}$, so $P[X = 2] = 0.3750$

Using Binomial Tables: Example 2

Ten percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 10 parts contains more than 3 defective ones? Use the binomial distribution table to answer this question.

Answer::

Using Binomial Tables: Example 2

Ten percent of computer parts produced by a certain supplier are defective. What is the probability that a sample of 10 parts contains more than 3 defective ones? Use the binomial distribution table to answer this question.

Answer:: Here $n = 10$, $p = \frac{10}{100} = 0.1$, so
$P[x > 3] = 1 - P[X \leq 3] = 1 - 0.9872 = 0.0128$

Using Binomial Tables: Example 3

Emily hits 60% of her free throws in basketball games. She had 25 free throws in last week games. What is the probability that she made at least 20 hits? Use the binomial distribution table to answer this question.

Answer:

Using Binomial Tables: Example 3

Emily hits 60% of her free throws in basketball games. She had 25 free throws in last week games. What is the probability that she made at least 20 hits? Use the binomial distribution table to answer this question.

Answer: Here $n = 25$, $p = 0.6$, so

$$\begin{aligned} P[X \geq 20] &= P[X = 20] + P[X = 21] + ... + P[X = 25] \\ &= 0.0199 + 0.0071 + 0.0019 + 0.0004 + 0.0000 + 0.0000 \\ &= 0.0293 \end{aligned}$$

Alternatively,

$$P[X \geq 20] = 1 - P[X < 20] = 1 - P[X \leq 19] = 1 - 0.9706 = 0.0294$$

Notice the small difference due to rounding.

The Geometric Distribution

Definition

When a random variable X represents the number of independent Bernoulli trials x, where the probability of success is p and probability of failure is q, until the 1^{st} success (including the success), we say that X has a ***Geometric distribution***. The probability of any number of successes x can be expressed as:

$$P[X = x] = q^{x-1} \times p$$

The expected value of X is $\mathbb{E}[X] = \frac{1}{p}$
The Variance of X is $\sigma^2 = \frac{q}{p^2}$

The Geometric Distribution: Example 1

Suppose we roll a coin until it lends on heads. Let X represent the number of coin flips required until it lands on heads. What is the probability that it lends on heads on the third roll? What is the Expected Value of X? What is the standard deviation of X

Answer::

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Suppose we roll a coin until it lends on heads. Let X represent the number of coin flips required until it lands on heads. What is the probability that it lends on heads on the third roll? What is the Expected Value of X? What is the standard deviation of X

Answer:: X follows a Geometric distribution.

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Answer:: X follows a Geometric distribution.

$$P[X = 3] = 0.5^{3-1} \times 0.5 = 0.125$$

The expected value of X is $\mathbb{E}[X] = \frac{1}{0.5} = 2$
The Variance of X is $\sigma^2 = \frac{0.5}{0.5^2} = 2$

The Geometric Distribution: Example 2

A couple who have a 20% chance of conceiving a girl, plan to keep having kids until they get a girl. Let X be the random variable representing the number of times they will need to try before getting a girl. What is the probability that they get a girl on the fifth attempt? What is the Expected Value of X? What is the variance of X

Answer::

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Answer:: X follows a Geometric distribution.

$$P[X = 5] = 0.8^{5-1} \times 0.2 = 0.08192$$

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The standard deviation of X is $\sigma = \sqrt{\frac{0.8}{0.2^2}} = \sqrt{20} = 4.472135955$

The Geometric Distribution: Example 3

Suppose that a six sided die is biased such that it only lands on even numbers 15% of the time. Let X represent the number of times we have to roll the die before it land on an even number. What is the probability that we do not flip an even number until the 3^{rd}, 5^{th}, and 7^{th} attempt? What is the Expected Value of X? What is the variance of X

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Answer:: X follows a Geometric distribution with $p = 0.15$ and $q = 0.85$.

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The standard deviation of X is $\sigma = \sqrt{\frac{0.85}{0.15^2}} = \sqrt{20} = 6.14636$

The Hypergeometric Probability Distribution

Definition

The hypergeometric distribution describes the probability of x successes in n draws, without replacement, from a population of size N that contains exactly k objects with the feature of interest. In contrast, the binomial distribution describes the probability of x successes in n draws with replacement, where $p = k/N$.

The Hypergeometric Prb. Dist., Contd.

So, in the hypergeometric distribution the trials are *dependent* and the probability of success *changes* from one trial to the next.

Definition, Contd.

Let N be the population sampled.
Let k the number of items in the population that are classified as successes.
Draw a sample of n individuals, without replacement.
Let X represent the number of items in the sample classified as successes.
We say X has a hypergeometric probability distribution $h(x; n; k; N)$, and

$$P[X = x] = \frac{C_x^k \times C_{n-x}^{N-k}}{C_n^N} \quad ; \mathbb{E}[X] = \frac{nk}{N} \quad ; \sigma^2 = \frac{nk(N-k)}{N^2} \times \frac{N-n}{N-1}$$

Hypergeometric Prob. Dist.: Example 1

Suppose a cooler has 20 cans of coke, C, and 10 of Pepsi, P. Suppose we draw five cans, without replacement. Let X be a random variable representing the number of coke cans. What is the probability distribution of X? What is $\mathbb{E}[X]$ and what is σ_X^2?

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$$P[X = 0] = \frac{C_0^{20} \times C_{5-0}^{30-20}}{C_5^{30}} = 0.0018$$

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Hypergeometric Prob. Dist.: Example 1, Contd.

$$P[X=3]=\frac{C_3^{20}\times C_{5-3}^{30-20}}{C_5^{30}}=0.3600$$

Hypergeometric Prob. Dist.: Example 1, Contd.

$$P[X = 3] = \frac{C_3^{20} \times C_{5-3}^{30-20}}{C_5^{30}} = 0.3600$$

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$$P[X = 5] = \frac{C_5^{20} \times C_{5-5}^{30-20}}{C_5^{30}} = 0.1088$$

Hypergeometric Prob. Dist.: Example 1, Contd.

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Hypergeometric Prob. Dist.: Example 1, Contd.

What is $\mathbb{E}[X]$ and what is σ_X^2?

Recall $N = 30$, $n = 5$, and $k = 20$.

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Recall $N = 30$, $n = 5$, and $k = 20$.

$$\mathbb{E}[X] = \frac{nk}{N} = \frac{5 * 20}{30} = 3.33$$

What is $\mathbb{E}[X]$ and what is σ_X^2?

Recall $N = 30$, $n = 5$, and $k = 20$.

$$\mathbb{E}[X] = \frac{nk}{N} = \frac{5 * 20}{30} = 3.33$$

$$\sigma^2 = \frac{nk(N-k)}{N^2} \times \frac{N-n}{N-1} = \frac{5 * 20(30-20)}{30^2} \times \frac{30-5}{30-1} = 0.95785$$

Hypergeometric Prob. Dist.: Example 1, Contd.

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Hypergeometric Prob. Dist.: Example 1. Continued

The table below summarizes the probability distribution of X.

x	$P[x]$
0	0.0018
1	0.0295
2	0.1600
3	0.3600
4	0.3400
5	0.1088

Hypergeometric Prob. Dist.: Example 2

A wallet contains 5 \$100 bills and 15 \$1 bills. You randomly choose 6 bills, without replacement. What is the probability that you will choose exactly 2 \$100 bills?

Answer:

Hypergeometric Prob. Dist.: Example 2

A wallet contains 5 \$100 bills and 15 \$1 bills. You randomly choose 6 bills, without replacement. What is the probability that you will choose exactly 2 \$100 bills?

Answer: Here $N = 20$, $n = 6$, and $k = 5$, so:

Hypergeometric Prob. Dist.: Example 2

A wallet contains 5 \$100 bills and 15 \$1 bills. You randomly choose 6 bills, without replacement. What is the probability that you will choose exactly 2 \$100 bills?

Answer: Here $N = 20$, $n = 6$, and $k = 5$, so:

$$P[X = 2] = \frac{C_2^5 \times C_{6-2}^{20-5}}{C_6^{20}} = \frac{C_2^5 \times C_4^{15}}{C_6^{20}} = 0.3522$$

Out of 8 students qualifying an exam, 6 are females. If 4 students were randomly drawn without replacement, find the probability that 3 females were chosen.

Answer:

Hypergeometric Prob. Dist.: Example 3

Out of 8 students qualifying an exam, 6 are females. If 4 students were randomly drawn without replacement, find the probability that 3 females were chosen.

Answer: here $N = 8$, $n = 4$, and $k = 6$, so:

Hypergeometric Prob. Dist.: Example 3

Out of 8 students qualifying an exam, 6 are females. If 4 students were randomly drawn without replacement, find the probability that 3 females were chosen.

Answer: here $N = 8$, $n = 4$, and $k = 6$, so:

$$P[X = 3] = \frac{C_3^6 \times C_{4-3}^{8-6}}{C_4^8} = 0.5714$$

Out of 50 light bulbs, 5 are defective. An inspector randomly samples 4 bulbs without replacement. Let X be the number of defective bulbs selected. Find the probability distribution of X.

Answer:

Hypergeometric Prob. Dist.: Example 4

Out of 50 light bulbs, 5 are defective. An inspector randomly samples 4 bulbs without replacement. Let X be the number of defective bulbs selected. Find the probability distribution of X.

Answer: here $N = 50$, $n = 4$, and $k = 5$. So:

Hypergeometric Prob. Dist.: Example 4

Out of 50 light bulbs, 5 are defective. An inspector randomly samples 4 bulbs without replacement. Let X be the number of defective bulbs selected. Find the probability distribution of X.

Answer: here $N = 50$, $n = 4$, and $k = 5$. So:

$$P[X = 0] = \frac{C_0^5 \times C_{4-0}^{50-5}}{C_4^{50}} = 0.6470$$

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Answer: here $N = 50$, $n = 4$, and $k = 5$. So:

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$$P[X = 1] = \frac{C_1^5 \times C_{4-1}^{50-5}}{C_4^{50}} = 0.3081$$

Hypergeometric Prob. Dist.: Example 4

Out of 50 light bulbs, 5 are defective. An inspector randomly samples 4 bulbs without replacement. Let X be the number of defective bulbs selected. Find the probability distribution of X.

Answer: here $N = 50$, $n = 4$, and $k = 5$. So:

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$$P[X = 3] = \frac{C_3^5 \times C_{4-3}^{50-5}}{C_4^{50}} = 0.0020$$

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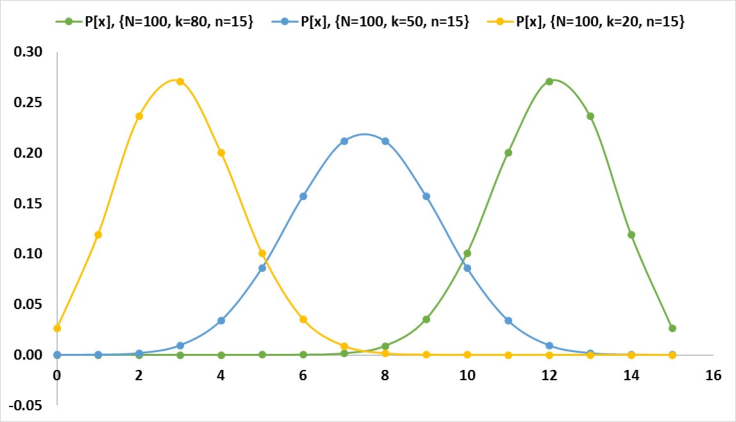
Shape of the Hypergeometric Prob. Dist.

Let $N = 100$, $n = 15$, and allow k to change:



Comparing the Binomial and Hypergeometric

Recall that the binomial distribution requires the trials to be independent with the probability of success being the same from trial to trial. When we sample without replacement, however, the trials become dependent and the probability of success changes each time we draw.

When the population size N is large compared to sample size n, then the hypergeometric distribution with parameters N, n, and k (which corresponds to sampling without replacement) is well approximated by the binomial distribution with parameters n and $p = \frac{k}{N}$ (which corresponds to sampling with replacement).

Comparing the Binomial and Hypergeometric

Suppose a random variable X follows a hypergeometric distribution with $N = 40$, $n = 5$, and $k = 8$. This could be approximated with a binomial distribution with $n = 5$ and $p = \frac{k}{N} = \frac{8}{40} = 0.2$. Use excel to compare the probability distribution of X using Binomial and Hypergeometric distributions.

Verify that:

$$P[X = x] = \frac{C_x^8 \times C_{5-x}^{32}}{C_5^{40}}$$
$$\approxeq C_x^5 \times (0.2)^x \times (0.8)^{5-x}$$

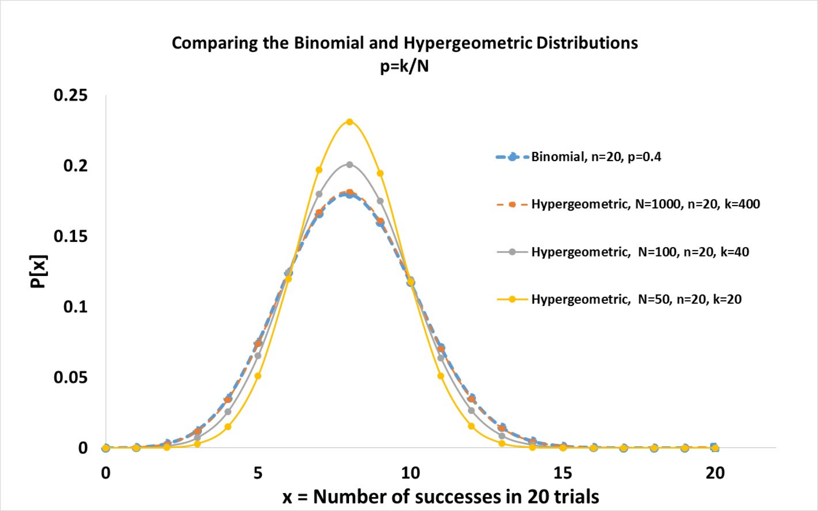
Comparing the Binomial and Hypergeometric: Continued

The probability distribution of X using Binomial and Hypergeometric distributions:

x	Hypergeometric: $P[x]$	Binomial: $P[x]$
0	0.3060	0.3277
1	0.4372	0.4096
2	0.2111	0.2048
3	0.0422	0.0512
4	0.0034	0.0064
5	0.0001	0.0003

Comparing Binomial and Hypergeometric

Suppose we fix $p = \frac{k}{N} =$ at 0.4 and n at 20.



Poisson Distribution

Definition

The ***Poisson distribution*** is a discrete probability distribution that is applied to events for which the probability of occurrence is calculated over a given span of time, space, or distance. The discrete random variable, X, is the number of times the event occurs over the given span.

$$P[X = x] = \frac{\lambda^x e^{-\lambda}}{x!}$$

$\lambda = \mathbb{E}[X]$ is the expected number of occurrences over the given span

$e = 2.71828$ is the base of the natural logarithm system.

$\mathbb{E}[X] = Var(X) = \lambda$

Poisson Distribution: Example 1

In an urban county, the average number of births is 1.2 babies per day. Let X represent the number of daily births. Find $P[X = 0]$, $P[X = 1]$, $P[X = 2]$, $P[X = 3]$, and $P[X = 4]$.

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$$P[X = 0] = \frac{1.2^0 \times e^{-1.2}}{0!} = 0.3012$$

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$$P[X = 3] = \frac{1.2^3 \times e^{-1.2}}{3!} = 0.0867$$

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$$P[X = 3] = \frac{1.2^3 \times e^{-1.2}}{3!} = 0.0867$$

$$P[X = 4] = \frac{1.2^4 \times e^{-1.2}}{4!} = 0.0260$$

Poisson Distribution: Example 1

In an urban county, the average number of births is 1.2 babies per day. Let X represent the number of daily births. Find $P[X = 0]$, $P[X = 1]$, $P[X = 2]$, $P[X = 3]$, and $P[X = 4]$.

$$P[X = 0] = \frac{1.2^0 \times e^{-1.2}}{0!} = 0.3012$$

$$P[X = 1] = \frac{1.2^1 \times e^{-1.2}}{1!} = 0.3614$$

$$P[X = 2] = \frac{1.2^2 \times e^{-1.2}}{2!} = 0.2169$$

$$P[X = 3] = \frac{1.2^3 \times e^{-1.2}}{3!} = 0.0867$$

$$P[X = 4] = \frac{1.2^4 \times e^{-1.2}}{4!} = 0.0260$$

Poisson Distribution: Example 2

A life insurance salesman sells on average 3 life insurance policies per week. Let X represent the number of policies sold. Find $P[X = 5]$, $P[X > 0]$, $P[2 \leq X < 5]$, $E[X]$, and σ_X

Answer:

Poisson Distribution: Example 2

A life insurance salesman sells on average 3 life insurance policies per week. Let X represent the number of policies sold. Find $P[X = 5]$, $P[X > 0]$, $P[2 \leq X < 5]$, $E[X]$, and σ_X

Answer:
Here $\lambda = 3$. $P[X = 5] = \frac{3^5 \times e^{-3}}{5!} = 0.1008$

Poisson Distribution: Example 2

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Answer:
Here $\lambda = 3$. $P[X = 5] = \frac{3^5 \times e^{-3}}{5!} = 0.1008$
$P[X > 0] = 1 - P[X \leq 0] = 1 - P[X = 0] = 1 - \frac{3^0 \times e^{-3}}{0!} = 1 - 0.0498 = 0.9502$

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$$\begin{aligned} P[2 \leq X < 5] &= P[X = 2] + P[X = 3] + P[X = 4] \\ &= \frac{3^2 \times e^{-3}}{2!} + \frac{3^3 \times e^{-3}}{3!} + \frac{3^4 \times e^{-3}}{4!} \\ &= 0.2240 + 0.2240 + 0.1680 = 0.6161 \end{aligned}$$

Poisson Distribution: Example 2

A life insurance salesman sells on average 3 life insurance policies per week. Let X represent the number of policies sold. Find $P[X = 5]$, $P[X > 0]$, $P[2 \leq X < 5]$, $E[X]$, and σ_X

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$$E[X] = \lambda = 3 \Rightarrow \sigma_X = \sqrt{3}$$

Poisson Distribution, Individual Probabilities
for x = number of occurrences, prob ($x = k$)

	λ	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
	1	0.3662	0.3614	0.3543	0.3452	0.3347	0.3230	0.3106	0.2975	0.2842	0.2707
	2	0.2014	0.2169	0.2303	0.2417	0.2510	0.2584	0.2640	0.2678	0.2700	0.2707
	3	0.0738	0.0867	0.0998	0.1128	0.1255	0.1378	0.1496	0.1607	0.1710	0.1804
	4	0.0203	0.0260	0.0324	0.0395	0.0471	0.0551	0.0636	0.0723	0.0812	0.0902
k	5	0.0045	0.0062	0.0084	0.0111	0.0141	0.0176	0.0216	0.0260	0.0309	0.0361
	6	0.0008	0.0012	0.0018	0.0026	0.0035	0.0047	0.0061	0.0078	0.0098	0.0120
	7	0.0001	0.0002	0.0003	0.0005	0.0008	0.0011	0.0015	0.0020	0.0027	0.0034
	8	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002	0.0003	0.0005	0.0006	0.0009
	9			0.0000	0.0000	0.0000	0.0000	0.0001	0.0001	0.0001	0.0002
	10							0.0000	0.0000	0.0000	0.0000

Poisson Distribution Tables: $P[X \leq x]$

Poisson Distribution, Cumulative Probabilities
for x = number of occurrences, prob ($x \leq k$)

	λ	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
	0	0.3329	0.3012	0.2725	0.2466	0.2231	0.2019	0.1827	0.1653	0.1496	0.1353
	1	0.6990	0.6626	0.6268	0.5918	0.5578	0.5249	0.4932	0.4628	0.4337	0.4060
	2	0.9004	0.8795	0.8571	0.8335	0.8088	0.7834	0.7572	0.7306	0.7037	0.6767
	3	0.9743	0.9662	0.9569	0.9463	0.9344	0.9212	0.9068	0.8913	0.8747	0.8571
	4	0.9946	0.9923	0.9893	0.9857	0.9814	0.9763	0.9704	0.9636	0.9559	0.9473
k	5	0.9990	0.9985	0.9978	0.9968	0.9955	0.9940	0.9920	0.9896	0.9868	0.9834
	6	0.9999	0.9997	0.9996	0.9994	0.9991	0.9987	0.9981	0.9974	0.9966	0.9955
	7	1.0000	1.0000	0.9999	0.9999	0.9998	0.9997	0.9996	0.9994	0.9992	0.9989
	8			1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9998	0.9998
	9							1.0000	1.0000	1.0000	1.0000

Using Poisson Tables: Example 1

The average number of homes sold by a realtor is 2 homes per day. What is the probability that exactly 3 homes will be sold tomorrow? Use a Poisson table to answer this question.

Answer:

Using Poisson Tables: Example 1

The average number of homes sold by a realtor is 2 homes per day. What is the probability that exactly 3 homes will be sold tomorrow? Use a Poisson table to answer this question.

Answer:
Here $\lambda = 2$, so

$$P[X = 3] = 0.1804$$

Using Poisson Tables: Example 2

Suppose the average number of lions seen on a 1-day safari is 5. What is the probability that tourists will see at most four lions on the next 1-day safari? Use a Poisson table to answer this question.

Answer:

Using Poisson Tables: Example 2

Suppose the average number of lions seen on a 1-day safari is 5. What is the probability that tourists will see at most four lions on the next 1-day safari? Use a Poisson table to answer this question.

Answer:
Here $\lambda = 5$, so

$$P[X \leq 4] = 0.4405$$

Using Poisson Tables: Example 3

The number of calls coming per hour into a hotel's reservation center is 9 on average. Let X be the number of calls received, and find $P[X = 0]$, $P[X = 5]$, $P[X = 15]$, $P[X = 22]$, $P[X \leq 7]$, $P[X \geq 17]$, $P[X > 10]$, $E[X]$, and σ_X^2. Use a Poisson table to answer these questions.

Answer:

Using Poisson Tables: Example 3

The number of calls coming per hour into a hotel's reservation center is 9 on average. Let X be the number of calls received, and find $P[X = 0]$, $P[X = 5]$, $P[X = 15]$, $P[X = 22]$, $P[X \leq 7]$, $P[X \geq 17]$, $P[X > 10]$, $E[X]$, and σ_X^2. Use a Poisson table to answer these questions.

Answer:
Here $\lambda = 9$, so

$P[X = 0] = 0.0001$

Using Poisson Tables: Example 3

The number of calls coming per hour into a hotel's reservation center is 9 on average. Let X be the number of calls received, and find $P[X = 0]$, $P[X = 5]$, $P[X = 15]$, $P[X = 22]$, $P[X \leq 7]$, $P[X \geq 17]$, $P[X > 10]$, $E[X]$, and σ_X^2. Use a Poisson table to answer these questions.

Answer:

Here $\lambda = 9$, so

$P[X = 0] = 0.0001$
$P[X = 5] = 0.0607$

Using Poisson Tables: Example 3

The number of calls coming per hour into a hotel's reservation center is 9 on average. Let X be the number of calls received, and find $P[X = 0]$, $P[X = 5]$, $P[X = 15]$, $P[X = 22]$, $P[X \leq 7]$, $P[X \geq 17]$, $P[X > 10]$, $E[X]$, and σ_X^2. Use a Poisson table to answer these questions.

Answer:
Here $\lambda = 9$, so

$P[X = 0] = 0.0001$
$P[X = 5] = 0.0607$
$P[X = 15] = 0.0194$

Using Poisson Tables: Example 3

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Answer:

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$P[X = 0] = 0.0001$
$P[X = 5] = 0.0607$
$P[X = 15] = 0.0194$
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Using Poisson Tables: Example 3

The number of calls coming per hour into a hotel's reservation center is 9 on average. Let X be the number of calls received, and find $P[X=0]$, $P[X=5]$, $P[X=15]$, $P[X=22]$, $P[X \leq 7]$, $P[X \geq 17]$, $P[X > 10]$, $E[X]$, and σ_X^2. Use a Poisson table to answer these questions.

Answer:

Here $\lambda = 9$, so

$P[X=0] = 0.0001$ $\qquad$ $P[X \leq 7] = 0.3239$

$P[X=5] = 0.0607$

$P[X=15] = 0.0194$

$P[X=22] = 0.0001$

Using Poisson Tables: Example 3

The number of calls coming per hour into a hotel's reservation center is 9 on average. Let X be the number of calls received, and find $P[X = 0]$, $P[X = 5]$, $P[X = 15]$, $P[X = 22]$, $P[X \leq 7]$, $P[X \geq 17]$, $P[X > 10]$, $E[X]$, and σ_X^2. Use a Poisson table to answer these questions.

Answer:
Here $\lambda = 9$, so

$P[X = 0] = 0.0001$
$P[X = 5] = 0.0607$
$P[X = 15] = 0.0194$
$P[X = 22] = 0.0001$

$P[X \leq 7] = 0.3239$
$P[X \geq 17] = 1 - P[X < 17] = 1 - P[X \leq 16] = 1 - 0.9889 = 0.0111$

Using Poisson Tables: Example 3

The number of calls coming per hour into a hotel's reservation center is 9 on average. Let X be the number of calls received, and find $P[X = 0]$, $P[X = 5]$, $P[X = 15]$, $P[X = 22]$, $P[X \leq 7]$, $P[X \geq 17]$, $P[X > 10]$, $E[X]$, and σ_X^2. Use a Poisson table to answer these questions.

Answer:
Here $\lambda = 9$, so

$P[X = 0] = 0.0001$	$P[X \leq 7] = 0.3239$
$P[X = 5] = 0.0607$	$P[X \geq 17] = 1 - P[X < 17] = 1 - P[X \leq 16] =$
$P[X = 15] = 0.0194$	$1 - 0.9889 = 0.0111$
$P[X = 22] = 0.0001$	$P[X > 10] = 1 - P[X \leq 10] = 1 - 0.7060 = 0.294.$

Using Poisson Tables: Example 3

The number of calls coming per hour into a hotel's reservation center is 9 on average. Let X be the number of calls received, and find $P[X = 0]$, $P[X = 5]$, $P[X = 15]$, $P[X = 22]$, $P[X \leq 7]$, $P[X \geq 17]$, $P[X > 10]$, $E[X]$, and σ_X^2. Use a Poisson table to answer these questions.

Answer:

Here $\lambda = 9$, so

$P[X = 0] = 0.0001$

$P[X = 5] = 0.0607$

$P[X = 15] = 0.0194$

$P[X = 22] = 0.0001$

$P[X \leq 7] = 0.3239$

$P[X \geq 17] = 1 - P[X < 17] = 1 - P[X \leq 16] = 1 - 0.9889 = 0.0111$

$P[X > 10] = 1 - P[X \leq 10] = 1 - 0.7060 = 0.294.$

$E[X] = \sigma_X^2 = \lambda = 9$

Approximating the Binomial distribution with Poisson

When n (the number of trials) is relatively large and p (the probability of success) is small, the binomial distribution can be closely approximated by the Poisson distribution.

As a rule of thumb, the binomial distribution can be satisfactorily approximated by the Poisson whenever $n \geq 20$ and $p \leq 0.05$. Under these conditions, we can just use $\lambda = np$ and find the probability of each value of X using the Poisson distribution.

Approximating the Binomial with Poisson: Example

Past experience has shown that 1% of the microchips produced by a certain firm are defective. A sample of 30 microchips is randomly selected from the firm's production. If X is the number of defective microchips in the sample, determine $P[X = 0]$, $P[X = 1]$, $P[X = 2]$, $P[X = 3]$, $P[X = 4]$, $P[X = 5]$, $P[X = 6]$. Use excel to solve this problem and compare the Binomial and Poisson distributions.

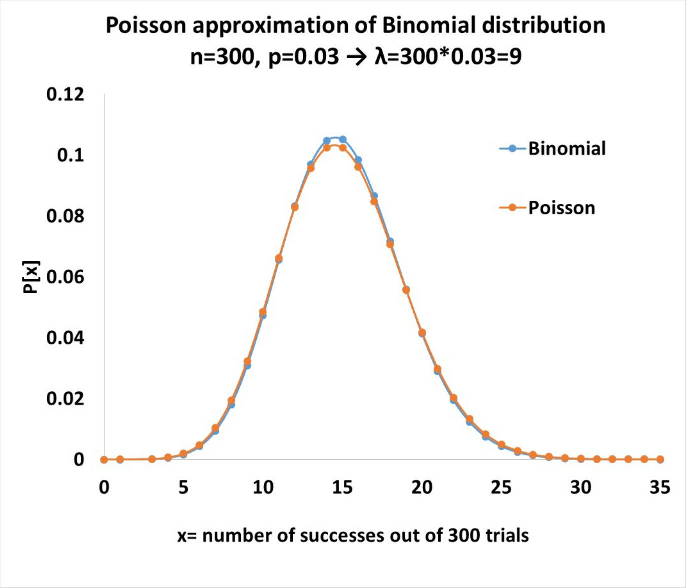
Notice that $n = 30 > 20$ and $p = 0.01 < 0.05$. So, $\lambda = 30 \times 0.01 = 0.3$

$$P[X = x] = C_x^{30} \times (0.01)^x \times (0.99)^{30-x}$$
$$\approxeq \frac{0.3^x \times e^{-0.3}}{x!}$$

Approximating the Binomial with Poisson: Continued

x	Binomial $P[x]$	Poisson $P[x]$
0	0.73970	0.74082
1	0.22415	0.22225
2	0.03283	0.03334
3	0.00310	0.00333
4	0.00021	0.00025
5	0.00001	0.00002
6	0.00000	0.00000

Approximating the Binomial with Poisson



Approximating the Binomial distribution with Poisson: Problem 1

Suppose that 4% of tires at a factory are defective. An inspector takes a random sample of 100 tires with replacement. Let X be the number of defective tires in the sample. What is the probability that more than 6 tires are defective?

Answer:

Approximating the Binomial distribution with Poisson: Problem 1

Suppose that 4% of tires at a factory are defective. An inspector takes a random sample of 100 tires with replacement. Let X be the number of defective tires in the sample. What is the probability that more than 6 tires are defective?

Answer:
Notice that $n = 100 > 20$ and $p = 0.04 < 0.05$. So, $\lambda = 100 \times 0.04 = 4$

$$P[X > 6] = 1 - P[X \leq 6] = 1 - 0.8893 = 0.1107$$

Approximating the Binomial distribution with Poisson: Problem 2

There is a 2% chance that a person tests positive for a certain virus. A lab receives a random sample of 375 patients from a large population for testing. What is the probability that at least 10 people test positive?

Answer:

Approximating the Binomial distribution with Poisson: Problem 2

There is a 2% chance that a person tests positive for a certain virus. A lab receives a random sample of 375 patients from a large population for testing. What is the probability that at least 10 people test positive?

Answer:
Notice that $n = 375 > 20$ and $p = 0.02 < 0.05$. So,
$\lambda = 375 \times 0.02 = 7.5$

$$P[X \geq 10] = 1 - P[X < 10] = 1 - P[X \leq 9] = 1 - 0.7764 = 0.2236$$