Homework assignment 10

1. For a population of five individuals, television ownership is as follows:

	x = Number of TVs Owned
Allen	2
Betty	1
Chuck	3
Dave	4
Eddie	2

(a) Determine the probability distribution for the discrete random variable, X = number of television sets owned. Calculate the population mean and standard deviation.

Answer:

x = Number of TVs Owned	$P[x]$
1	0.2
2	0.4
3	0.2
4	0.2

$$\mu = \sum P[x_i] * x_i = 2.4$$

$$\sigma^2 = \sum P[x_i] * [x_i - \mu]^2 = 1.04$$

(b) For the sample size $n = 2$, determine the mean for each possible simple random sample from the five individuals.

(c) For each simple random sample identified in part (b), what is the probability that this particular sample will be selected?

Answer:

sample	$\bar{x}$ = mean of this sample	probability of selecting this sample
Allen, Betty	$(2 + 1)/2 = 1.5$	1/10
Allen, Chuck	$(2 + 3)/2 = 2.5$	1/10
Allen, Dave	$(2 + 4)/2 = 3.0$	1/10
Allen, Eddie	$(2 + 2)/2 = 2.0$	1/10
Betty, Chuck	$(1 + 3)/2 = 2.0$	1/10
Betty, Dave	$(1 + 4)/2 = 2.5$	1/10
Betty, Eddie	$(1 + 2)/2 = 1.5$	1/10
Chuck, Dave	$(3 + 4)/2 = 3.5$	1/10
Chuck, Eddie	$(3 + 2)/2 = 2.5$	1/10
Dave, Eddie	$(4 + 2)/2 = 3.0$	1/10

2. A random variable is normally distributed with mean $\mu = \$1500$ and standard deviation $\sigma = \$100$. Determine the standard error of the sampling distribution of the mean for simple random samples with the following sample sizes: $n = 16$, $n = 100$, $n = 400$, and $n = 1000$.

   Answer: for $n = 16$, the standard error is

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{100}{\sqrt{16}} = 25$$

   You can get the standard error for $n = 100$, $n = 400$, and $n = 1000$ in a similar manner.

3. For a random variable that is normally distributed, with mean $\mu = 200$ and standard deviation $\sigma = 20$ determine the probability that a simple random sample of 4 items will have a mean that is:(a) greater than 210, (b) between 190 and 230, and (c) less than 225.

Answer: Since $X \sim \mathcal{N}(200, 20)$, $\bar{X} \sim \mathcal{N}(200, \frac{20}{\sqrt{4}} = 10)$

$$\begin{aligned} P[\bar{X} > 210] &= P[\frac{\bar{X} - 200}{10} > \frac{210 - 200}{10}] \\ &= P[z > 1] = 1 - P[z < 1] = 0.1587 \end{aligned}$$

$$\begin{aligned} P[190 < \bar{X} < 230] &= P[\frac{190 - 200}{10} < \frac{\bar{X} - 200}{10} < \frac{230 - 200}{10}] \\ &= P[-1 < z < 3] = 0.8400 \end{aligned}$$

$$\begin{aligned} P[\bar{X} < 225] &= P[\frac{\bar{X} - 200}{10} < \frac{225 - 200}{10}] \\ &= P[z < 2.5] = 0.9938 \end{aligned}$$

4. Shirley Johnson, a county political leader, is looking ahead to next year's election campaign and the possibility that she might be forced to support a property tax increase to help combat projected shortfalls in the county budget. For the dual purpose of gaining voter support and finding out more about what her constituents think about taxes and other important issues, she is planning to hold a town meeting with a simple random sample of 40 homeowners from her county. For the more than 100,000 homes in Shirley's county, the mean value is \$190,000 and the standard deviation is \$50,000. What is the probability that the mean value of the homes owned by those invited to her meeting will be greater than \$200,000?

   Answer: $n > 30$ so $\bar{X} \sim \mathcal{N}(19000, \frac{50000}{\sqrt{40}} = 7905.69)$, so

$$P[\bar{X} > 200000] = P[z > 1.26] = 0.1038$$

5. It has been reported that the average U.S. teenager sends 80 text messages per day. For purposes of this exercise, we will assume the daily number of text messages sent is normally distributed with a standard deviation of 15 messages. For a randomly selected group of 10 teenagers, and considering these persons to be a simple random sample of all U.S. teens, what is the probability that the group will send at least 900 text messages (i.e., a sample mean of at least 90 messages per teen) next Wednesday?

Answer: Since $X \sim \mathcal{N}(80, 15)$, $\bar{X} \sim \mathcal{N}(80, \frac{15}{\sqrt{10}} = 4.743)$

$$\begin{aligned} P[\bar{X} > 90] &= P[z > \frac{80 - 90}{4.743}] \\ &= P[z > 2.11] = 1 - P[z < 2.11] = 0.0174 \end{aligned}$$