Homework assignment 6b

1. A city law-enforcement official has stated that 20% of the items sold by pawn shops within the city have been stolen. Ralph has just purchased 4 items from one of the city's pawn shops. Assuming that the official is correct, and for $x =$ the number of Ralph's purchases that have been stolen, determine the following: $P[x = 0]$, $P[2 \leq x]$, $P[1 \leq x \leq 3]$, and $P[x \leq 2]$. What is $\mathbb{E}[x]$ and what is the standard deviation of x.

   Answer: Using the binomial function: $P[x] = C_x^n p^x q^{n-x}$, with $n = 4$ and $p = 0.2$, we can calculate:

$$P[x = 0] = C_0^4 p^0 q^{4-0} = \frac{4!}{0!(4-0)!} 0.2^0 0.8^{4-0} = 0.4096$$

$$P[x = 1] = C_1^4 p^1 q^{4-1} = \frac{4!}{1!(4-1)!} 0.2^1 0.8^{4-1} = 0.4096$$

$$P[x = 2] = C_2^4 p^2 q^{4-2} = \frac{4!}{2!(4-2)!} 0.2^2 0.8^{4-2} = 0.1536$$

$$P[x = 3] = C_3^4 p^3 q^{4-3} = \frac{4!}{3!(4-3)!} 0.2^3 0.8^{4-3} = 0.0256$$

$$P[x = 4] = C_4^4 p^4 q^{4-4} = \frac{4!}{4!(4-4)!} 0.2^4 0.8^{4-4} = 0.0016$$

   So,

$$P[2 \leq x] = P[x = 2] + P[x = 3] + P[x = 4] = 0.1808$$

$$P[1 \leq x \leq 3] = P[x = 1] + P[x = 2] + P[x = 3] = 0.5888$$

$$P[x \leq 2] = P[x = 0] + P[x = 1] + P[x = 2] = 0.9728$$

   The expected value is $\mathbb{E}[x] = 0.2 * 0.4 = 0.8$, and $\sigma = \sqrt{0.2 * 0.8 * 4} = 0.8$.

2. Each child born to a particular set of parents has probability 0.25 of having blood type O. Suppose these parents have 5 children, let x represent the number of these children who have blood type O. Calculate the probability of each value x can take. What is the mean and standard deviation of x.

   Answer: This is a binomial distribution with $p = 0.25$, $n = 5$. See the table below.

x	$P[x]$
0	0.2373
1	0.3955
2	0.2637
3	0.0879
4	0.0146
5	0.0010

   $\mathbb{E}[x] = n * p = 5 * 0.25 = 1.25$, and $\sigma = \sqrt{0.25 * 0.75 * 5} = 0.968$.

3. Suppose we have a four-child family. Each child may be either a boy (B) or a girl (G). For simplicity we suppose that $P[B] = P[G] = \frac{1}{2}$ and that the genders of the children are determined independently. Let x represent the number of girls in this family. What is $P[x = 3]$? What is the mean and standard deviation of x?

   Answer: This is a binomial distribution with $p = \frac{1}{2}$, $n = 4$. So:. $P[x = 3] = C_3^4(\frac{1}{2})^3(\frac{1}{2})^{4-3} = 0.25$, $\mathbb{E}[x] = 4 * \frac{1}{2} = 2$, and $\sigma^2 = 4 * \frac{1}{2} * \frac{1}{2} = 1$. Alternatively, you can use the probability distribution table below:

x	$P[x]$	$\sum P[x_i]x_i$	$\sum P[x_i](x_i - \mathbb{E}[x])^2$
0	0.0625	0.0000	0.2500
1	0.2500	0.2500	0.2500
2	0.3750	0.7500	0.0000
3	0.2500	0.7500	0.2500
4	0.0625	0.2500	0.2500
		2.0	1

4. Suppose the chance of winning a lottery is $\frac{1}{1000}$. Suppose you bought 10 lottery tickets. Let x be the number of winning tickets in your purchased tickets. What is the probability that you have at least one winning ticket? What is the probability that you have no wining tickets? What is the probability that all your tickets are winning tickets.

   Answer: x takes on the values 0, 1, 2, $\ldots$, 10. The purpose of this question was to make you think and provide approximate answers. The probability that you have at least one winning ticket is practically zero.

$$\begin{aligned} P[x \geq 1] &= P[x = 1] + P[x = 2] + \cdots + P[x = 10] \\ &= 0.009910 + 0.000045 + 0 + 0 + \cdots + 0 = 0.009955 \end{aligned}$$

   On the other hand, the probability that you have no winning tickets is $P[x = 0] = 0.990045$, which means it's almost certain to lose this lottery.

   The probability that all your tickets are winning tickets is virtually zero. To be exact, $P[x = 10] = 1 * 10^{-30}$