Homework assignment 9: Solution

1. The following data values are a simple random sample from a population that is normally distributed, with $\sigma^2 = 25.0 : \{47, 43, 33, 42, 34, 41\}$. Construct and interpret the 95% and 99% confidence intervals for the population mean.

   Answer: First, verify that $\bar{x} = 40$, notice that $n = 6$. Since the population standard deviation is known $\sigma = \sqrt{25} = 5$,

   $$\mu \in [\bar{x} \pm z * \frac{\sigma}{\sqrt{n}}]$$

   (a)

   $$\mu \in [40 \pm 1.96 * \frac{5}{\sqrt{6}}] \text{ simplify and interpret this.}$$

   (b)

   $$\mu \in [40 \pm 2.58 * \frac{5}{\sqrt{6}}] \text{ simplify and interpret this.}$$

2. A simple random sample of 25 has been collected from a normally distributed population for which it is known that $\sigma = 17.0$. The sample mean has been calculated as 342.0, and the sample standard deviation is $s = 14.9$. Construct and interpret the 95% and 99% confidence intervals for the population mean.

   Answer: $\bar{x} = 342$, $n = 25$. Since the population standard deviation is known $\sigma = 17$,

   $$\mu \in [\bar{x} \pm z * \frac{\sigma}{\sqrt{n}}]$$

   (a)

$$\mu \in [342 \pm 1.96 * \frac{17}{\sqrt{25}}] \text{ simplify and interpret this.}$$

(b)

$$\mu \in [342 \pm 2.58 * \frac{17}{\sqrt{25}}] \text{ simplify and interpret this.}$$

3. For $df = 25$, determine the value of A that corresponds to each of the following probabilities: (a) $P[t \geq A] = 0.025$, (b) $P[t \leq A] = 0.1$, (c) $P[-A \leq t \leq A] = 0.99$

   Answer: Remember that the t Table shows the area to the right of A. With $df = 25$,

   a $P[t \geq A] = 0.025 = 2.060$. This is from the 0.025 column and the $df = 25$ row of the t Table. There was a typo in the Homework for this part.

   b To solve $P[t \leq A] = 0.1$, first solve $P[t \geq A] = 0.1$ which produces a t score of $t = 1.316$. This is from the 0.1 column and the $df = 25$ row of the t Table. Since the curve is symmetrical, the value of t for a left tail area associated with $P[t \leq A] = 0.1$ is $A = 1.316$.

   c To solve $P[-A \leq t \leq A] = 0.99$, notice that the 0.01 of the total area left out will be evenly split between the two tails. Each tail will have an area of $\alpha = \frac{1-0.99}{2} = 0.005$. Referring to the 0.005 column and the $df = 25$ row of the t Table, $A = 2.787$.

4. A consumer magazine has contacted a simple random sample of 33 owners of a certain model of automobile and asked each owner how many defects had to be corrected within the first 2 months of ownership. The average number of defects was $\bar{x} = 3.7$, with a standard deviation of $s = 1.8$ defects.

   Answer: $n = 33$, $\bar{x} = 3.7$, $s = 1.8$, and σ is unknown, so

$$\mu \in [\bar{x} \pm t * \frac{s}{\sqrt{n}}]$$

   - Use the t distribution to construct a 95% confidence interval for μ, the average number of defects for this model.

$\alpha = \frac{1-0.95}{2} = 0.025$, $df = 32$, so $t = \pm 2.037$, and

$$\mu \in \left[3.7 \pm 2.037 * \frac{1.8}{\sqrt{33}}\right] \text{ simplify and interpret this.}$$

- Use the z distribution to construct a 95% confidence interval for μ, the average number of defects for this model.

  For a confidence level of 95%, $z = 1.96$, so:

$$\mu \in \left[3.7 \pm 1.96 * \frac{1.8}{\sqrt{33}}\right] \text{ simplify and interpret this.}$$

- Given that the population standard deviation is not known, which of these two confidence intervals should be used as the interval estimate for μ?

  If σ is not known, the t distribution should be used in constructing a 95% confidence interval for μ. Therefore, the confidence interval found in the first section is the correct one.

5. An automobile rental agency has the following mileages (in thousands) for a simple random sample of 20 cars rented last year. Given this information, and assuming the data are from a population that is approximately normally distributed, construct and interpret the 90% confidence interval for the population mean: $\{55, 35, 65, 64, 69, 37, 88, 80, 39, 61, 54, 50, 74, 92, 59, 50, 38, 59, 29, 60\}$

   Answer: With $n = 20$, verify that $\bar{X} = 57.90$, and $s = 17.38$. Since σ is unknown, $\mu \in [\bar{x} \pm t * \frac{s}{\sqrt{n}}]$. $\alpha = \frac{1-0.90}{2} = 0.05$, $df = 19$, so $t = \pm 1.729$, and

$$\mu \in [57.90 \pm 1.729 * \frac{17.38}{\sqrt{20}}] \text{ simplify and interpret this.}$$

6. The average capacity usage for iPhone users has been estimated as 400 megabytes per month. Assuming this finding to be based on a simple random sample of 80 iPhone users, with a sample standard deviation of $s = 90$ megabytes per month, construct and interpret the 95% confidence interval for the population mean usage per month. Given this confidence interval, would it seem very unusual if another sample of this size were to have a mean of 350.0 megabytes per month?

Answer: $n = 80$, $\bar{X} = 400$, and $s = 90$. Since σ is unknown, $\mu \in [\bar{x} \pm t * \frac{s}{\sqrt{n}}]$. $\alpha = \frac{1-0.95}{2} = 0.025$, $df = 89$, so $t = \pm 1.990$, and

$$\mu \in [400 \pm 1.990 * \frac{90}{\sqrt{80}}] \text{ simplify and interpret this.}$$

7. In response to media inquiries and concerns expressed by groups opposed to violence, the president of a university with over 25,000 students has agreed to survey a simple random sample of her students to find out whether the student body thinks the school's "Plundering Pirate" mascot should be changed to one that is less aggressive in name and appearance. Of the 200 students selected for participation in the survey, only 20% believe the school should select a new and more kindly mascot. Construct a 90% confidence interval for the population proportion of students who believe the mascot should be changed. Based on the sample findings and associated confidence interval, comment on the credibility of a local journalist's comment that "over 50%" of the students would like a new mascot.

   $\hat{p} = 0.2$, $n = 200$ so,

$$p \in \left[\hat{p} \pm z * \sqrt{\frac{\hat{p}\hat{q}}{n}}\right]$$

$$p \in \left[0.2 \pm 1.645 * \sqrt{\frac{0.2 * 0.8}{200}}\right] = [0.153, 0.247]$$

   So, we are 90% confident that the the percentage of students who would like a new mascot is in the interval $[0.153, 0.247]$. Based on this confidence interval, the 0.5 value falls far above the upper limit, and it would not seem credible that "over 50% of the students would like a new mascot.

8. A study by Careerbuilder.com found that 20% of companies check out job candidates' profiles on social networking sites like Facebook and MySpace before deciding whether to employ them. Assuming that the survey included a simple random sample of 1200 companies, construct a 90% confidence interval for p, the population proportion of companies that check social-networking sites before offering employment.

$\hat{p} = 0.2$, $n = 1200$ so,

$$p \in \left[\hat{p} \pm z * \sqrt{\frac{\hat{p}\hat{q}}{n}} \right]$$

$$p \in \left[0.2 \pm 1.645 * \sqrt{\frac{0.2 * 0.8}{1200}} \right] \text{ simplify and interpret this.}$$